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Recent Advances in Applications and Its Real-world Implementations of Graph Theory

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ABSTRACT

The field of mathematics plays a vital role in various fields. One of the most important areas in mathematics is Graph theory. It is a sophisticated mathematical framework for studying the properties and connections of graphs that allows data to be imaged and helps to simplify the complex structure of data with sets of vertices and edges. The significant advances have been made in graph theory and its applications in recent years, leading to novel techniques and real-world implementations. This summary provides an overview of these recent developments. The emergence of Graphical Neural Networks is a significant development that enables them to learn graph-structured data representations. The GNNs have been successfully applied in various domains, including node classification and link prediction. The community detection algorithms have also improved accuracy and efficiency. It will benefit social network analysis and other related fields. Graph embedding techniques have also progressed, facilitating learning low-dimensional vector representations for nodes or subgraphs. This advancement has enhanced tasks such as link prediction and node classification. Furthermore, the temporal graph analysis has gained attention for studying dynamic graphs, allowing for predicting future states and anomaly detection. The application of graph theory in social networks has yielded valuable insights into sentiment dynamics, influence maximization, and information diffusion. These findings aid to understand the social behavior's and designing effective interventions. Network alignment and graph matching techniques have been developed to integrate and analyze data from multiple sources. The finding applications in diverse fields, including biological networks and cross-domain data analysis. Graph theory has significantly contributed to transportation and urban planning, addressing challenges like traffic flow optimization, route planning, and public transportation design. Furthermore, the application of graph theory in bioinformatics and drug discovery has led to advancements in drug-target interaction and protein function prediction. Recent advances in graph theory have opened up new avenues for research and practical applications. The growing availability of large-scale Graph data and the need to extract insights from complex interconnected systems continue the evolution of this field. These developments have great potential for addressing real-world challenges in various domains.

Keywords - Directed Graph, Bipartite graph, Binary Tree, Vertex Colouring, Community Detection, Influence Maximization, Network Embedding, Temporal Graphs, Graph Databases.

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I. INTRODUCTION

Graphs are important because a graph is a way of expressing information in pictorial form. Graph theory is becoming significant because it is applied to other areas of mathematics and various branches of science and technology. It is widely used in various fields, such as biochemistry, chemistry, computer science, engineering, operations research and real life. They are often used in studying and modelling various applications. Graphs are used to model many problems of the real world in various fields. Diagrams are considered very powerful and a flexible tool to model problems. Applications in the computer industry makes extensive use of the principles of graph theory, like data mining, image segmentation, clustering, image capture, networking, etc., are the active areas of study in computer science. For example, a tree-shaped data system will use nodes and connections to organize its data. Network topologies can be modelled using graph ideas. Both resource allocation and optimization, organization use the central idea of graph colour.

In Graph theory, the best applications include traveling sales persons or seller's complexity, database design principles, resource networking, use of routes, styles, and circuits. This results in the creation of sophisticated algorithms and theorems with far reaching practical implications. For many situations in the real world, a diagram can be described simply by adding several nodes and edges connecting the several pairs of these points. Points may refer to romantic partners or people with lines depicting other types of relationships between them and the outside world with close friends or contact centers. In such graphs, the way to represent the lines are connected is secondary to whether or not they connect two designated places. Graph theory provides a statistical summary of such situations and the concepts are widely used in many fields for analysis and simulation. The study of atoms, molecules and chemical bonding are all fall under this umbrella. Graph theory has many applications in the social sciences, including the study of networks and the study of how ideas spread from one place to another. The theory of Graphs is used to protect biodiversity, with peaks representing habitats and edges representing movement paths between habitats. This information is important for studying the spread of diseases and parasites and the impact of migration on other species. We know the graph theory concepts are abundant in the field of computing, such as Width to Search, Depth to Search, Topological Order, Belman-Ford, Dijkstra's algorithm, Minimalist trees, Kruskal's algorithm, and Prim's algorithm are all examples of algorithms used in Graph theory.

II. APPLICATIONS OF GRAPH THEORY

The ideas of graph theory are widely used to study and simulate various applications. This encompasses the study of atoms, molecules, and chemical bonding. Graph theory, also has applications in the social sciences, such as studying diffusion processes and determining a performer's fame. In biology and conservation, diagrams show where different species live and where their migrations and other movements take them. This information is vital for studying migratory patterns, tracking the spread of illnesses and parasites, and understanding the impact of migration on other species [4] and [5]. Principles of theoretical graphics are often used in scientific investigations. Finds the shortest path between two nodes and solves problems like the tour salesperson's conundrum and the little stretch in a weighted network. Other applications include simulating transportation systems, business networks, and game theory [6]. The finite game approach is represented as a digraph. In this case, the vertices indicate the places and the edges of the paths taken. Graph theory has several applications in science and engineering. Given some of the following:

2.1 LINGUISTICS:

In linguistics, graphs are often used for the parsing of language trees and grammar for language trees. Lexical semantics uses semantic networks, which are very useful for computers, and makes it easier to model the meaning of words by interpreting them in context. As a diagrammatic tool, phonological analysis, such as optimum theory and morphological analysis, such as finite-state morphology using finite state transducers are widely used in the study of languages.

2.2 PHYSICAL SCIENCES*:*

To study molecule in Chemistry, graphs represent a substance's molecular and chemical structures. It can also be used to construct the molecular structure and lattice of any molecule. It also helps in showing the bonding relationship between atoms and molecules, it also comparing the structure of one molecule to another. Here atoms can be considered as vertices of a graph and the bond that connects them is represented as edges between them. These structures are created based on the properties of compounds and are taken for analysis and processing. This can be used to study the structure of the molecules and to check the similarity level between the molecules. With the help of graphs, the 3D structure of complicated simulated atomic structures can be studied quantitatively by the gathering data on graph theoretic properties related to atomic topology.

In physics, a salient role in electrical modelling of electrical networks. Current, voltage and resistance in a circuit can be designed using the concept of Graph theory. To show the flow of current in circuits we can use directed graphs. Graphs can represent local connections between interacting parts of a system in statistical physics, as well as the dynamics of a physical process on such systems. To build circuit linkages. Topologies are used to describe these kinds of connections. Examples include sequence topology, bridge topology, star topology, and parallel topology.

In physics and chemistry, molecules are analyzed using graph theory. Statistics on graph theoretic properties in connection to atom topology allow for quantitative analysis of the 3D layout of complicated artificial atomic systems. Statistical mechanics also make use of graphs. In this field, diagrams depict the local connections between the interacting parts of a system and the dynamics of the underlying physical processes. Porous media microchannels may be expressed as graphs, with the vertices representing the larger pores and the boundaries representing the smaller pores. The molecular structure and molecular grid may benefit from using a graph. It also enables us to compare the structures of different molecules and show how atoms are connected to them.

2.3.BIOLOGICAL SCIENCES*:*

Graph Theory is used in many areas of Biology. It can be used in drug target identification, determining a protein's function, gene's function. It is also used in studying the structures of DNA and RNA. Graphs are used to represent relationships among species on different physical and micro-biological criteria. For example, the evolutionary relationships among the existing species are expressed in a tree structure called phylogenetic tree.

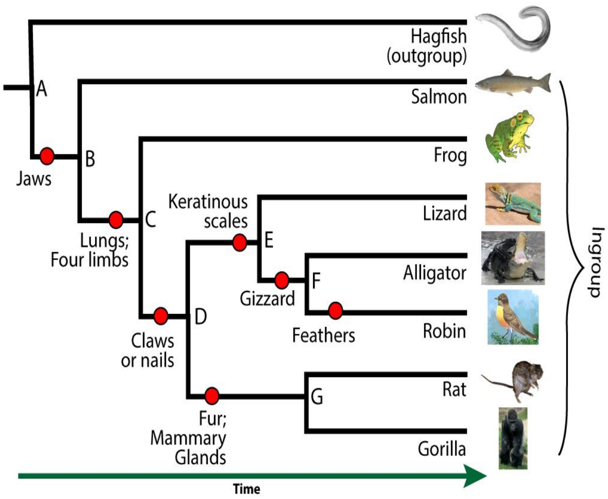

Figure. 1

Graphs are also used in analyzing biological data. Ecological landscapes can be modelled using graphs. Habitat patches are represented as vertices and the movement between the patches is represented as edges when ecological landscapes are modelled as graphs. Similarly, Graph theory is useful in conservation efforts where a vertex represents region where certain species exist and the edges represent migration paths or movement between the regions. This information is important when tracking the spread of diseases, parasites and how the changes in the movement can affect other species.

Biomolecular entities, such as chromosomes, proteins, or metabolites serve as the "nodes" in biological networks, while the "edges" connecting the nodes represent the interactive, physical, or chemical interactions between the nodes. It is used for transcriptional regulation networks, and it can see in metabolic networks. Protein-Protein Interaction (PPI) networks are another use of graph theory. Partnerships that share a drug-related objective. Synergistic Effects of Drugs.

2.4. OPERATIONS RESEARCH:

Graph Theory is a very useful tool in Operations research. Some problems in Operations Research make use of graphs, which makes it easier to solve the problem. Modelling, transportation, network activity, game theory, minimum cost path and the scheduling problem are some examples of applications of graph theory in operations research.

A large number of combinatorial problems are solved using network activity. The planning and scheduling of large complex projects are one of the most popular and successful applications of networks in operations research. Project Evaluation Review Technique (PERT) and Critical Path Method (CPM) are two of the most well know problems using Graph theory.

Graph theory, is also used in different assignment and transportation problems, such as assigning different people to different jobs, scheduling time tables and also in maximal flow problems. Transportation problem is a directed graph application, where each edge has a weight and each edge receives a flow, where the amount of flow cannot exceed the capacity of the edge. In transportation problem, when we need to minimize the transportation cost or maximize the profit, then the graph theoretical approach is very useful. Here directed graph is called a network, the vertices are called nodes and the edges are known as arcs. To find the best way to perform a specific task in competitive situations, game theory is applied to problems in engineering, economics, war science, etc. In this case, vertices represent the positions and the edges represent the movements.

2.5. COMPUTER SCIENCE:

Graph theoretical concepts have wide scope in computer science areas such as website designing, network security, communication network and so on. A data structure can be designed in the form of a tree which in turn utilized vertices and edges. Similarly modelling of network topologies can be done using graph concepts. Graphs are used to describe the computation flow. Graph transformation systems use in memory graph manipulation based on rules. Graph databases allow for the safe transaction, persistent storage and querying of graph-structured data.

The theory is used in computer graphics to examine algorithms like the Dijkstra Algorithm, the Prims Algorithm, and the Kruskal Algorithm. The application domains used to define the computation flow include graphs. Communication webs may be represented using graphs. Graphs illustrate the structure of the findings. Rules based graph manipulation is the foundation for graph transformation methods. Graph databases provide reliable, always-available storage and querying for structured graph data. The fastest path across a network may be determined with the help of graph theory. In Google Maps, locations are represented as vertices or points, while corners represent roads; this representation is used to calculate the shortest route between any two given points.

2.6. WEBSITE DESIGNING*:*

Graphs can be used to design websites. Google's successful web search algorithms are based on the WWW graph, in which the web pages are represented by vertices and hyperlinks between them are represented by the edges in the graph. Here complete bipartite graph plays a vital role that each vertex representing a type of object is connected to every vertex representing other kind of objects. There are many advantages of using graph representation in website development such as searching and community discovery. Structure of a websites containing many pages can be represented using a directed graph. Each page can be considered as a vertex. A link exists between two pages if there exists an edge between them. By this way, we can identify which page is accessible from which page.

2.7. NETWORK SECURITY AND COMMUNICATION NETWORK:

Graphs are used to simulate the propagation of stealth worms on large computer networks and to develop strategies to protect the networks from virus attacks. For this the French Navy ESCANSIC used the vertex cover algorithm. The idea is to find an optimal solution for

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designing the network design strategy. The main aim is to find a minimum vertex cover in the graph whose vertices are the route servers and whose edges are the connection between the routing servers.

It can be used to represent communication networks, which is a collection of terminals, links and nodes which enables communication between users of terminals. Every communication network has three basic components namely terminals, processors and transmission channels. This network transmits packets of data between computers, telephones or other devices. Graph theory helps to model the communication network by vertices as terminals, processors and edges represent transmission channels through which data flows. Thus, the data packets can be transmitted from input to output through a sequence of switches joined by directed edges. Communication network can be represented as a complete binary tree in which squares are represented as terminals, sources or destinations for packets of data, circles as switches that directs the packets through network. There is a unique path between every pair of vertices in an undirected tree, so that the switch can receive packets and forward in the computer binary tree in an analogue directed path.

Group Special Mobile (GSM) is a mobile phone network where the geographical area of this network is divided into hexagonal regions or cells. Each cell has a communication tower which connects with mobile phones within the region. All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four different frequency ranges, it is clear by the concept of graph theory that only four colours can be used to colour the cellular regions. These four different colours are used for proper colouring of the regions. Therefore, the vertex colouring algorithm may be used to assign at most four frequencies for any GSM mobile phone network.

The concept of assigning the colours is as follows: In a Graph drawn on the plane or on the surface of a sphere, the four-color theorem assets that it is always possible to colour the regions of a Graph properly using at most four distinct colours such that no two adjacent regions are assigned the same colour. Now, a dual graph is constructed by putting a vertex inside each region of the Graph and connect two distinct vertices by an edge if and only if their respective regions share a whole segment of their boundaries in common. Then proper colouring of the dual graph gives proper colouring of the original Graph. Since, colouring the regions of planar graph G is equivalent to colouring the vertices of its dual graph and vice versa. By colouring the Graph regions using four colour theorems, the four frequencies can be assigned to the regions accordingly.

2.8. SOCIAL SCIENCES:

The field of sociology also makes use of graph theory. For instance, social network analysis methods may probe how rumors spread or how credible specific individuals are. Their friendship and knowledge graphs describe the connectivity between people. Certain people can influence the actions of others in impactful diagrams. Using the collaborative graphs approach, two people work together in a familiar setting, such as watching a movie.

Sociology makes extensive use of graph theory. For example, Friendship and knowledge graphs describe whether individuals know each other or not. Social network analysis software to investigate rumor spreading or to assess actors' reputations. Using the influence graphs model to influence the behaviors of others by certain individuals. Collaboration graphs are used to examine whether two people collaborate in a special way, such as working in team together. Sociograms are used to represent relationships between people in a society or group, where a sociogram is a digraph that represents a person's social connections. In a sociogram, vertices denote people and directed edges denote relationships.

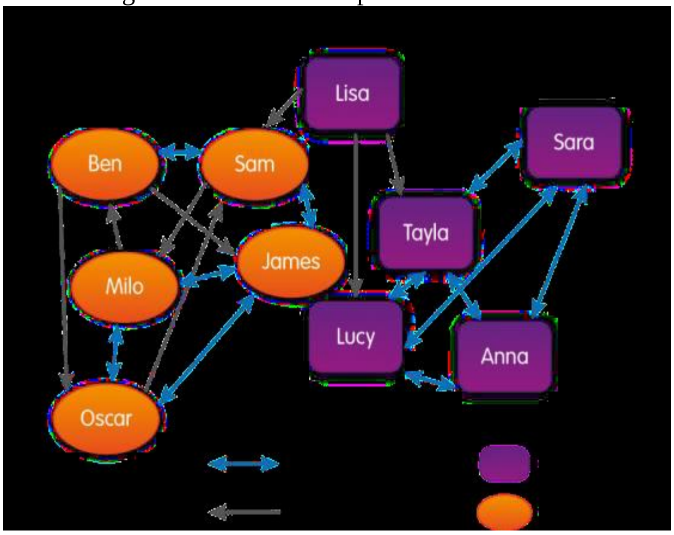

Figure. 2

Anthropologists have studied a number of tribes and classified them based on their association structures.

III. PROBLEMS IN GRAPH

3.1 ENUMERATION:

Graphical listing, or the issue of how many graphs there are that satisfy the stated requirements, has generated a large body of research. Harary and Palmer [6] present some of the results of this study. Sub-graphs, induced sub-graphs, and minors A common issue has a fixed graph as a subgraph in a given graph, called the subgraph isomorphism issue. A justification for being concerned is that specific graph properties for subgraphs are inherited, implying that a graph only owns the proprietorship if other subgraphs have the value. Sadly, it is always an NP-complete question to locate maximal subgraphs of a certain kind. Examples include:

- Clique problems have the maximum complexity and are thus NP-complete.
- Graph isomorphism is a subset of the broader subgraph isomorphism issue. It raises doubts about the isomorphism of the two diagrams. It is possible that we do not know whether this issue is NP-complete or if it can be solved in polynomial time.
- Similarly, mediated segments of a particular graph are identified. Again, some essential graph properties for induced subgraphs are inherited,

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implying that a map has a property only if they are all caused by all the caused subgraphs. Also, NP-complete is always the maximum mediated subgraphs of a certain kind. For instance:

- A different collection (NP-Complete) problem is the most edgeless influenced subgraph or isolated group.
- Another problem, the minor confinement issue, is having a fixed graph as a minor of a specific diagram. Every graph generated by taking a subgraph and contracting a few (or no) edges are a minor or subcontract. Most charts are passed to minors, meaning a character is only held if all minors share it. Wagner's Theorem states, for instance:
- A portion of the network is planar as it does not include the complete two-part map K3,3 or the complete map K5 as a minor.
- A related challenge is considering a defined diagram as a subset of a specified map, the challenge of a containment subset. Every graph generated by subdividing (or not) edges is a subset or homomorphism of a graph. The isolation of subdivisions is related to properties like planarity. For example, the theorem Kuratowski states:
- A diagram is flat because it does not contain a complete division K3,3 or a complete diagram K5 as a section.
- The Kelman's-Seymour assumption is another problem with unit containment:
- A subset of a 5-vertex graph K5 is used with a 5-vertex graph that is not planar.
- Another type of issue is linked to how many species are defined from their point-deleted subgraphs and to generalizations of maps. For instance: Speculation on reconstruction

3.2 GRAPH COLOURING:

Several problems of graph theory and theorems apply to different colouring strategies of graphs. In general, you are interested in painting a graph such that there are no two neighboring vertices with identical hues. Coloured edges (possibly such that no two edges are the same colour) or other differences can also be seen. The following are some of the famous findings and conclusions regarding graph colouring:

- The Erds-Faber-Lovász conjecture is still open, as is the Four-Color Theorem and the Strong Perfect Graph Theorem.
- Behzad's hypothesis (unresolved) on total coloring.
- The (unresolved) List Coloring Conjecture
- The unanswered Hadwiger hypothesis in graph theory

3.3 SUBSUMPTION AND UNIFICATION:

The theory of restricted design refers to families of graphs connected to a limited sequence. In such applications, graphs are organized so that more narrow graphs are subdivided into more general, more complex graphs, which contain more details. Machine graph integration and evaluating the route of a subsumption connection between two graphs are examples of graph operations. Effective unification methods are devised, and the most generic graph, i.e. incorporating all information in) the inputs, is defined as the result of unifying two argument graphs. Without the actual graph, just a single graph description is given. Graph fusion adequately fulfils and combines functions for restrictive structures that are purely compositional. Automatic hypothesis checking and modelling the creation of language structure are well-known applications.

3.4 ROUTE PROBLEMS:

- Problems like the Hamiltonian path issue, minimum spanning tree, and the route inspection problem sometimes known as the "Chinese postman problem" are all examples.
- The Steiner tree, the Three-Castle Problem, the Travelling Salesman Problem, and the Seven Bridges of Königsberg are all examples of NP-hard problems.

3.5 NETWORK FLOW:

Systems of diverse conceptions of network flows are facing multiple difficulties, for instance:

- Maximal Flow Minimum Cut Theorem

  Visibility problems:
- Issues with Museum Security

  Covering problems:

  Graphic coverage can apply to specific set coverage problems on vertical / subgraphs subsets.
- The prevailing issue in package covers is the unique case where the collection is closed.
- The problem with the cover vertex is the case with the frame cover, where the sets are all sides.
- The initial set cover issue can be represented as a vertex cover in a hypergraph, often called a hitting set.

3.6. DECOMPOSITION PROBLEMS:

There is a wide variety of concerns related to decomposition, defined as partitioning a graph's edge set (the number of vertices following each part's edges, as required). This is also required when subdividing a whole diagram into smaller sub diagrams, such as Hamiltonian cycles, that are isomorphic to the original. For example, many issues require decomposing a Kn network into n1 specified trees with 1, 2, 3..., n1 edges, where n is the number of edges in the graph. Among the many decomposition issues that have been examined are the following:

- Arboricity, decomposition to the fewest available trees
- Double loop wrapping, decomposition into loops that are precisely two-fold on each side
- Layer painting, decomposing in the fewest practicable matches

Factorization of the line, decomposition of an ordinary line into average graphs

IV. CONCLUSION

The purpose of this paper is to present the various importance and uses of graph theory ideas in different areas and topics of Linguistics, Physical and Biological sciences, Operations Research, Computer Science and applications, Communication, Web Designing, Network Security and Social Science, etc., This paper is designed to acquire the knowledge and benefits of the students of different disciplines to gain in depth knowledge on Graph Theory and its related areas and subjects. This paper focused on the various applications of major graph theory that have relevance to the field of computer science and applications.

REFERENCES

[1] West DB. Introduction to graph theory. Volume 2. Upper Saddle River: Prentice Hall; 2001

[2] Barabási A-L. Network science. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences. 2013;371:20120375

[3] Robinson Chelladurai and S.J. Maghy, Graph Theory in Different Networks, International journal of mathematics and its applications, Volume 6(1-D), 711 – 717.2018.

[4] Bhagya Jyoti Nath, Applications of Graph Theory in Different Branches of Science, International journal of mathematics and its applications, Volume 5, Issue 3-A, 57 – 60, 2017.

[5] J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Department of Combinatorics and Optimization, Univer-sity of Waterloo, Canada, 1976.

[6] Newman, Mark. Networks: An Introduction. Oxford University Press, 2010.

[7] Application of graph theory in communication networks by sumandeswal and Anita singhrova, International Journal of Application or Innovation in Engineering & Management, Volume 1, Issue 2, 2012.

[8] Rani, N.S., Suman, S.P., The role of data structure in multiple disciples in computer science. International Journal of Scientific & Engineering Research, 4(7), 2013.

[9] Durga Prasad, D.; Sneha Divya, M.; Kavitha, S. Applications of Computer Science Based on Graph theory. International Journal of Engineering Science (6) 2017.

[10] Liu, Y.; Safavi, T.; Dighe, A.; Koutra, D. Graph Summarization Methods and Applications: A Survey. ACM Computer Sur-vey. (51) 2018.

[11] Senthilraj. S, Graph Characterizations in Prime Square Domination, International Journal of Engineering Research & Management. Technology. ISSN: 2348-4039 vol 5, 2018.

[12] Li, P.; Yu, J.; Liu, J.; Zhou, D.; Cao, B. Generating weighted social networks using multigraph. Phys. A Stat. Mech. Its Appl. (539) 2020.

[13] Senthilraj S, Applications of B Method for Software Design Unsing Discrete Mathematics, International Journal of Discrete Mathematicical Sciences and Cryptography,Eashwar Publications Ltd., Vol 1, No 2 Pp:29-38, 2008.

[14] Senthilraj S, A Study on Biological Network Functional Model Designs using Domination Techniques in Graph Theory, ISSN: 2790-8259, Vol.1(1), 2022.