A Note on Public Key Cryptosystems

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ABSTRACT

Public key or asymmetric cryptosystems use public-private key pair for the secure transmission of data. RSA and ECC (Elliptic Curve Cryptography/Cryptosystems) are widely used cryptosystems in this category. Public key cryptosystems rely on mathematical problems known as hard problems. The security of these cryptosystems is based on these hard problems. Public key cryptosystems solve the key transportation problem of symmetric key cryptosystems and able to provides digital signatures also.

Keywords: **RSA, ECC (Elliptic Curve Cryptography/Cryptosystems), Security, Symmetric Key, Public Key**

INTRODUCTION:

Public key or asymmetric key cryptosystems use – public-private key pair. The data is encrypted by receiver's public key and can be decrypted by receiver's private key. Since the private key is never transmitted anywhere during the entire communication, public key cryptosystems provide high level of security. Data communication is a necessity of life in modern world so the importance of public key cryptosystems increases very sharply not only in cryptography and its allied areas [1-9] but also in wireless communication scenarios [10-15] and network security applications [16-20]. In cryptography, all cryptosystems can be divided into two categories. First is symmetric key cryptosystems and second is public key cryptosystems. In symmetric key cryptosystems, both sender and receiver use the same secret key for securing the data. The following figures show the difference between the above mentioned cryptosystems.

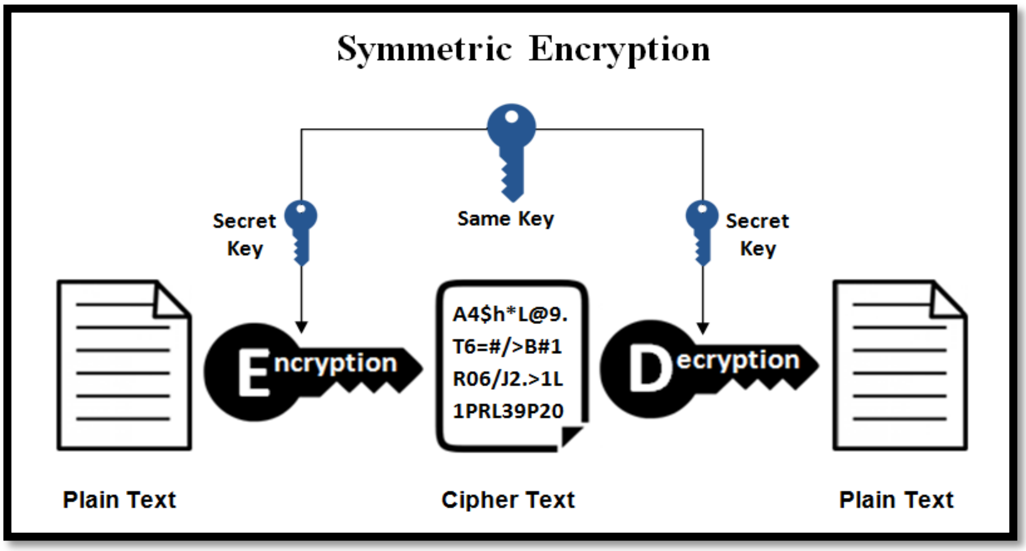


Fig.1. Showing Symmetric key encryption and decryption using the same key

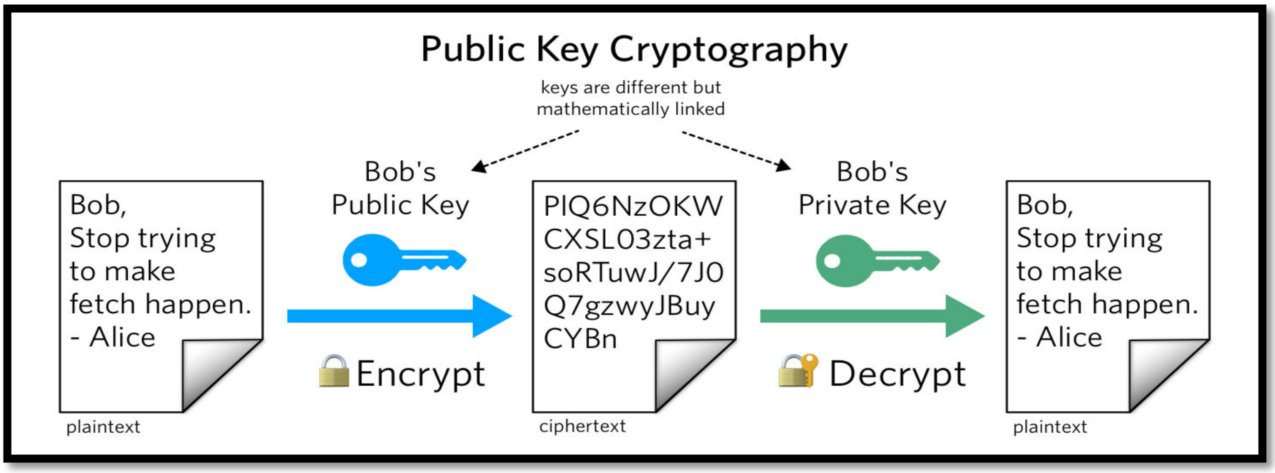
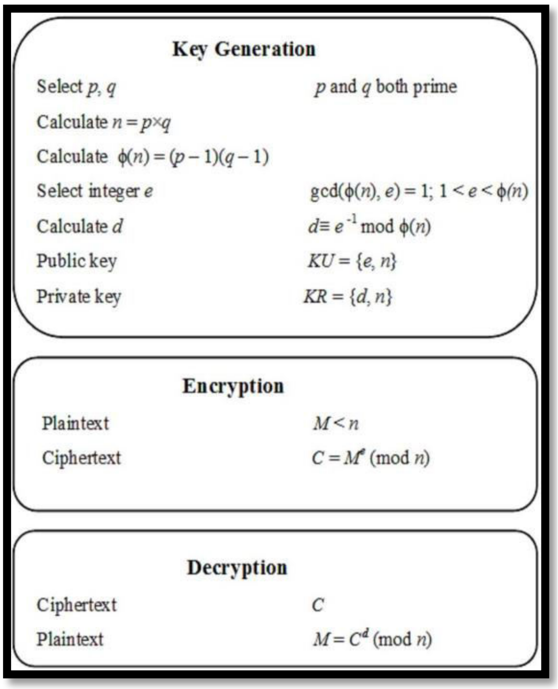

Public Key Cryptography
keys are different but mathematically linked
Bob's Public Key
Bob's Private Key
Bob, Stop trying to make fetch happen. - Alice
plaintext
Encrypt
PIQ6NzOKW CXSL03zta+ soRTuwJ/7J0 Q7gzwyJBuy CYBn
ciphertext
Decrypt
Bob, Stop trying to make fetch happen. - Alice
plaintext

Fig.2. Showing public key encryption and decryption using the key pair

Now we discuss some of the famous public key cryptosystems [21-22].

- **RSA**: RSA is the most trusted asymmetric key cryptosystem. It is based on prime factorization problem. One needs to select two prime numbers p and q so that n=pq. It is important to mention that p and q should be large enough for security. The full RSA method is described in the below figure for better understanding.



Key Generation

Select p, q	p and q both prime
Calculate $n = p \times q$	
Calculate $\phi(n) = (p-1)(q-1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \bmod \phi(n)$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

Encryption

Plaintext	$M < n$
Ciphertext	$C = M^e \pmod{n}$

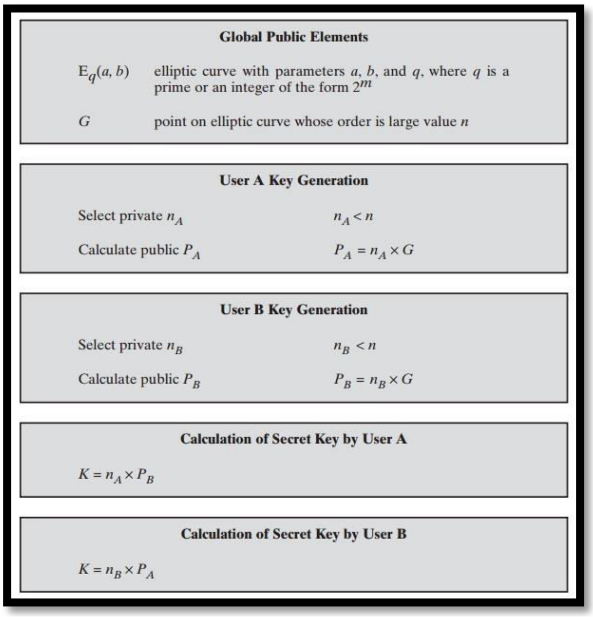
Decryption

Ciphertext	C
Plaintext	$M = C^d \pmod{n}$

Fig. 3. Showing the complete RSA procedure

- **Advantages:**
  - Easily implementable on various platforms
  - Highly secure as it is based on prime factorization
  - Widely accepted and trusted
- **Disadvantages:**
- Security is dependent on selection of prime numbers
- High computational overheads
- Public keys must be authenticated by a third party

- **ECC**: ECC provides equivalent level of encryption as provided by RSA with a shorter key length so naturally the speed of ECC is also faster than RSA. RSA-1024 bits key length security can be achieved by 160-223 bits key security of ECC. The basics of key generation in ECC are defined in the figure below.



Global Public Elements

$E_q(a, b)$	elliptic curve with parameters a, b, and q, where q is a prime or an integer of the form 2^m
G	point on elliptic curve whose order is large value n

User A Key Generation

Select private n_A	$n_A < n$
Calculate public P_A	$P_A = n_A \times G$

User B Key Generation

Select private n_B	$n_B < n$
Calculate public P_B	$P_B = n_B \times G$

Calculation of Secret Key by User A

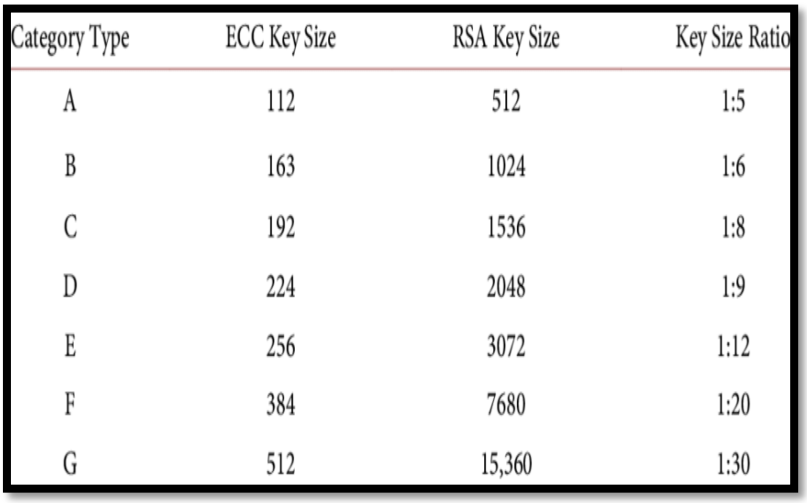
$K = n_A \times P_B$

Calculation of Secret Key by User B

$K = n_B \times P_A$

Fig.4. Showing the key generation in ECC

The key size comparison of ECC and RSA is given in the below figure.



Category Type	ECC Key Size	RSA Key Size	Key Size Ratio
A	112	512	1:5
B	163	1024	1:6
C	192	1536	1:8
D	224	2048	1:9
E	256	3072	1:12
F	384	7680	1:20
G	512	15,360	1:30

Fig.5. Showing the key size comparison between RSA and ECC

- **Advantages**:
  - ➢ Shorter key size
  - ➢ High level of security
  - ➢ Easily implementable
- **Disadvantages**:
  - ➢ Size of encrypted message is increased
  - ➢ Complex algorithm
  - ➢ Not widely accepted as RSA

CONCLUSION:

It can be concluded that public key cryptosystems use public-private key pair and solve the key transportation problem of symmetric key cryptosystems. The message is encrypted using the public key and private key is always kept secret by the participating entities. RSA and ECC are two main public key cryptosystems. RSA is the most trusted and widely accepted public key cryptosystem till date but uses large key for high level of security. On the other side, ECC provides equivalent level of security with reduced key size but not as trusted and widely accepted as RSA.

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