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Finding the optimum Roll-Off rate of a Raised Cosine Filter at the Receiver to Achieve the Lowest Bit Error Rate for a given Modulation Order and a given [(Energy/Bit) / Normalized Noise] Ratio

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Abstract

In the present age of digital communication, two parameters are of paramount importance- volume of data and the associated Bit Error Rate (BER).MIMO technique is a big help here. However, the phenomenon of multipath signals leads to Inter Symbol Interference (ISI). The pre-shaping of the pulse at the transmitter mitigates the ISI. Interestingly, the BER is dependent on the modulation order, the E_b/N_0 ratio and the roll-off rate of the filters. In this paper, we have studied the effects of the three mentioned parameters on the BER for a given system. The results do show some trends.

Key-words: Bit Error Rate (BER), Inter Symbol Interference (ISI), Multiple Input Multiple Output (MIMO), roll-off rate, Square Root Raised Cosine (SRRC).

1. INTRODUCTION

In wireless communication, the radio frequency channels play a most vital role to ensure the quality of service (QoS). The channels are usually characterized by two parameters- a limited bandwidth and a transfer function. The transfer function helps to know about two variable functions. One is the variable attenuation of different frequency components leading to attenuation distortion. The second one is the varying delay introduced for different paths- it gives rise to delay distortion.

These integral problems are caused partly by the physical properties of the medium and partly by the lack of perfection in the design of the transmit filters and the receive filters. A practical example is the radio channel, in which the transmitted signal reaches the receiver along multiple paths through reflections, diffractions and dispersion. The delayed components are equivalent to echoes that cause time dispersion of the transmitted signal.

If time dispersion is greater than a pre-set fraction of the signaling period, the channel responses to the different data signals overlap. This effect is called the inter symbol interference (ISI). As a consequence, the signal observed at the receiver

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input in such a condition is a superimposition of a certain number of data signals. Sometimes, the channel impulse response can span multiple signaling periods. Therefore, ISI is a major impairment introduced by the channel[1].

There have been lots of researches to counter the ISI problem. The ISI issue has been attacked at both the transmitter and at the receiver ends.

As has been seen, the delay involved in receiving a pulse from the source plays a very important role in determining the ISI, which leads to BER performance [2,5].

2. Theory

A Raised Cosine Filter (RCF), also known as a Raised Cosine Pulse-Shaping Filter, is a commonly used filter in digital communication systems to reduce Inter-Symbol Interference (ISI). Raised Cosine Filters are designed to mitigate ISI by modulating the transmitted signal in a specific way.

To demonstrate the improvement in Bit Error Rate (BER) when pulse shaping is applied using a Square Root Raised Cosine (SRRC) filter, we can compare the BER equations for the cases with and without pulse shaping. Pulse shaping using an SRRC filter is commonly used in digital communication systems to reduce the effects of inter symbol interference (ISI). The improvement is due to the fact that the SRRC filter helps to shape the transmitted pulses and tries to equalize the transmitted pulse and the received pulse mitigating ISI[1,5].

Let us represent the symbols transmitted over a communication channel as "s(t)" and the received signal as "r(t)." The received signal "r(t)" can therefore be expressed as a convolution of the transmitted symbols and the channel impulse response "h(t)". The resulting equation will be:

$$r(t)=s(t))*h(t) \ldots\ldots\ldots\ldots\ldots(1)$$

Without pulse shaping, the transmitted symbols are typically represented as rectangular pulses with duration Ts (symbol duration), which can be represented as:

$$p(t) = \mathrm{rect}\,(t/Ts)\ldots\ldots\ldots\ldots\ldots(2)$$

The received signal, in this case, will be:

$$r(t)=p(t)*h(t)+n(t)\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where:

- "*" denotes convolution.
- "n(t)" represents additive white Gaussian noise (AWGN).

Now, let's assume one is using SRRC pulse shaping with a roll-off factor of α. The roll-off factor shouldn't be too high as then the spectral bandwidth (BW) is lower and spectral energy at the higher ends of the spectrum can cause severe ISI. On the other hand, a lower value spreads the BW and as a result, the interference with the adjoining symbol periods is less. Usually α = 0.5 is a common choice. The SRRC pulse shape *p*SRRC(*t*) can be represented in the frequency domain as the Fourier transform of the SRRC pulse shape. This pulse shape spreads the BW but the spectral energy tapers down at the higher harmonics. It reduces ISI.

The received signal with SRRC pulse shaping can be expressed as:

$$rSRRC(t) = pSRRC(t)*h(t)+n(t) \ldots\ldots\ldots(4)$$

To evaluate the improvement in BER, we have evaluated the BER for various modulation orders, Eb/No and roll-off rates.

The actual BER equations can be quite complex and depend on the modulation scheme used and the specifics of the communication system. BER analysis often involves Monte Carlo simulations.

In summary, the improvement in BER when using SRRC pulse shaping is due to the reduction of ISI, which results from the shaping of transmitted pulses to better match the channel characteristics. This reduction in ISI leads to fewer errors in symbol detection and, consequently, a lower BER. However, the detailed BER analysis would require specific system parameters and simulation.

A very important parameter in this scheme is the Roll-Off factor (β) of the filter. It indicates the selectivity of the filter and is related to the spectral bandwidth. With the spreading of the spectral BW, the time domain signal generates more ripples and they tend to overlap with time periods allocated for

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the succeeding symbols. It leads to increase in ISI and, therefore, higher BER.

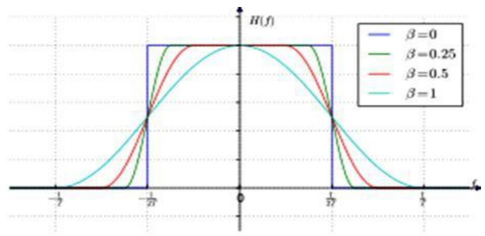

Figure 1: Variation of the shape of the spectrum with roll-off factor β

Figure 1 shows the typical characteristics of a frequency selective filter. The roll-off factor determines the rate of variation of the output with change in the input frequency. At lower values of β, the pulse shaping is wider and spectral BW is greater. Hence the spectral energy is lower at the higher ends of the spectrum. It ensures reduced ISI and lower BER.

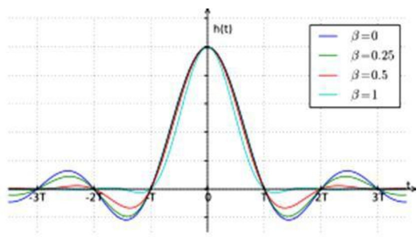

Figure 2: The diagram shows the time domain waveform for the shaped pulse for different β

As can be seen in the Fig 2, the signal dies down faster for high β. It means that the energy content for the signal portion beyond the defined symbol time is high for higher β since the spreading of the spectrum is lower. On the other hand, in Raised Cosine filtering, the technique is to spread the BW to reduce interference.

Additionally, the BER can be quantified more precisely using mathematical models, such as the Q-function for QAM modulation, and simulations that consider the specific α value, SNR, and channel characteristics. These models and simulations would provide BER curves that show how the BER changes with varying α values and SNR levels, allowing you to make informed design decisions for your communication system.

In summary, pulse shaping plays a vital role in reducing the Bit Error Rate by optimizing the use of available bandwidth, reducing ISI, mitigating noise, and ensuring compliance with communication standards. These benefits collectively enhance the reliability and performance of digital communication systems, making them better equipped to transmit data accurately in challenging environments like multi-path signaling as in MIMO systems.

3.Experiment and discussion of results

We have experimented with different combinations of QAM modulation order, [(Energy/Bit) / Normalized noise density] ((E_b/N_0) or SNR values) within the accepted range for reliable digital communication, roll-off rate to find out the variations of BER as per theory. We used MATLAB simulator.

We first examined the pattern of change in the Bit Error Rate (BER) with change in the value of E_b/N_0 ratio. This E_b/N_0 ratio is directly proportional to Signal-to-Noise ratio (SNR) and therefore, BER should be inversely proportional to E_b/N_0.

We carried out the experiment in three phases:

a) Modulation order versus BER, keeping roll-off rate and E_b/N_0 fixed.
b) BER variation with Eb/N0, keeping the roll-off rate and the modulation order fixed.
c) Roll-off rate versus BER keeping modulation order and E_b/N_0 fixed.

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Table 1: Readings obtained for the different combinations

Modulation Order	BER (Eb/No = 10dB and rolloff = 0.25)	Eb/No (in dB)	BER (Modulation Order= 16 and rolloff = 0.25)	Roll off factor	BER (Modulation Order= 16 and Eb/No = 10dB)
4	0	6	0.026853707	0.2	0.001903808
8	0.000450653	7	0.0174 8497	0.3	0.001803607
16	0.002154309	8	0.009068136	0.4	0.001703407
32	0.009774436	9	0.004308617	0.5	0.001503006
64	0.025621741	10	0.001002004	0.6	0.001853707
128	0.051013242	11	0.000701403	0.7	0.001903808
256	0.078564257	12	0.000250501	0.8	0.002254509
		13	5.01E-05	0.9	0.001753507
		14	0	1	0.001903808
		15	0		
		16	0		
		17	0		
		18	0		

a) Modulation order versus BER, keeping roll-off rate and E_b/N_0 fixed:

Fig. 3 depicts the variation of BER with varying modulation order. The order has been intentionally selected over a wide range – 4 to 256. It means the number of bits to represent a sample for a given alphabet may vary from a low 2 to a high of 8. It is seen that the BER for an order of 8 is the highest and decreases as the modulation order is lowered.

Let us discuss the results obtained. For modulation order 4, the result obtained is an aberration bur for other modulation orders, the BER pattern can be explained. When the order is high, the number of bits per sample is higher and the overall number of bits handled is also higher. It logically produces more inter symbol interference. Moreover, since the number of bits is high, the energy per bit is lower. It leads to degradation in the value of E_b/N_0 or SNR value. As we know, the BER is inversely proportional to the E_b/N_0 value and hence, BER value worsens at higher modulation order even at a reasonable value of 10dB for E_b/N_0. This trend will improve at rather high value of E_b/N_0, which may not be a practical solution.

b)BER variation with Eb/N0, keeping the roll-off rate and the modulation order fixed:

The E_b/N_0 ratio value is the most critical while comparing the performance of a radio receiver in data communication mode. Unless, the ratio exceeds some threshold value for a given network, the receiver fails to decode the binary files reliably. Therefore, we varied the value for E_b/N_0 from a healthy 18dB down to a very poor 6dB. For cellular networks, the value is kept around 18dB. However with introduction of equalizers and matching filters, the systems work very well even at 15dB- an improvement of 3dB.

It was observed that the BER falls regularly with increase in the E_b/N_0 value – we observed and plotted the values between 6dB and 18dB for E_b/N_0. The BER became negligible for values of E_b/N_0 greater than 14dB. So, if high value can be ensured in a radio network, the reliability of data communication will definitely improve. Fig. 4 shows the plot of BER variation with E_b/N_0.

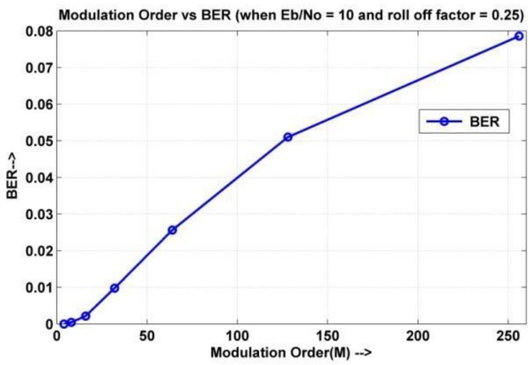

Figure 3: BER variation with Modulation order

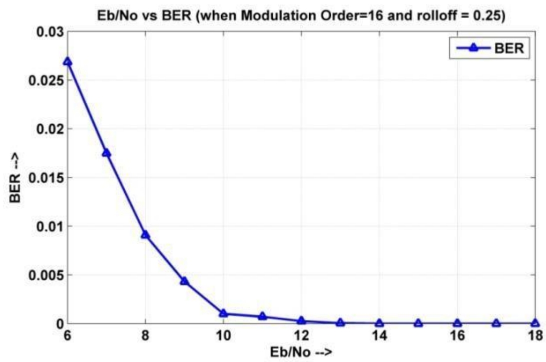

Figure 4: BER variation with E_b/N_0

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c)Roll-off rate and its effect on BER:

Mathematically and analytically, the BER should be low when the roll-off factor is high. It has been already explained in section 2. However, the relationship is not always linear because of a number of factors. The most important one is the Channel Impulse Response (CIR) and it is a highly time-dependent variable. In MATLAB simulator, the CIRs are generated randomly using Monte Carlo technique. Hence the reading for BER also changes even when the E_b /N_0 ratio is kept the same. As is seen from the curve plotted with the values of BER at different roll-off factors, the nature follows the expected behavior sometimes but not all the time.

For example, the BER falls gradually as the roll-off factor increases from 0.2 to 0.5 and it is not expected. After this, the BER jumps for 0.6 and 0.7 and, 0.8. It is expected but again, for the roll-off factor of 0.9, the BER drops sharply.

The consistency expected for high volume, high rate data can be achieved only if the correct type of equalizer is applied and here, Artificial Intelligence (AI) can play an important role.

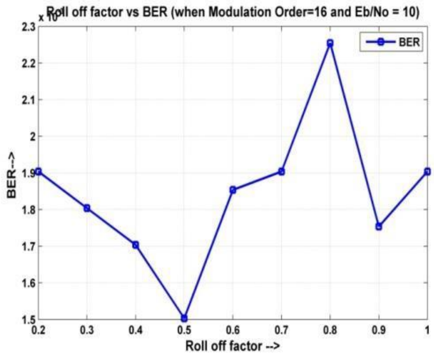

Figure 5: BER variation with Roll-Off rate

4.Conclusion and Future Work

We have been able to establish the close relation between the Pulse shaping filter shape in time domain and the Bit Error Rate. It, as we analysed, is related to the spread of the signal in the frequency domain which is responsible for the degree of interference with the succeeding signals. The intensity of interference controls the BER.

We have also seen that the BER is affected by the modulation order.

The compound effect of the modulation order, Roll-Off rate and the E_b/ N_0 ratio on the BER can be seen readily from the Eye diagram. We will carry on extensive experiments in this line and we expect to bring out some useful results soon.

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