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Sparseness Controlled Proportionate RLS Algorithm for Sparse and Non-Sparse Systems

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ABSTRACT

This paper proposes a novel sparse adaptive technique to handle variable sparsity. The algorithm is based on the measure of sparseness. A proportionate matrix distributes the gain factor for all filter taps during each iteration. Each filter coefficient of the adaptive filter is updated by the corresponding diagonal element of the proportionate matrix, and that depends on the degree of sparseness. The classical recursive least square algorithm (RLS) is amended by accommodating the proportionate matrix to propose the sparseness-controlled proportionate recursive least square (SC-PRLS) algorithm. The convergence control parameter is incorporated into the algorithm to achieve faster convergence and better steady-state error. The performance of Mean squared error and stead state error of the proposed algorithm are also compared with standard RLS and proportionate RLS (PRLS). The simulation results indicate that SC-PRLS is more effective than PRLS and traditional RLS. An increase in the degree of sparseness leads to an increase in steady-state error and it can be controlled by convergence control parameter, while the convergence rate remains intact in the SC-PRLS. It performs superior in sparse as well as in non-sparse environment so this algorithm can handle large variations in the sparseness.
Keywords: SC-PRLS, Sparseness Controlled Proportionate Adaptive Algorithm, Sparse and Non-Sparse Systems, RLS.

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1. INTRODUCTION

The classical adaptive digital signal processing algorithms do not perform well for the problems with sparse distribution like acoustic echo cancellation, optical equalizers, underwater acoustic communication, network echo cancellation, sparse system identification, etc. Recently, several variants of the classical algorithms were proposed to make use of sparsity leading to improvement in the performance for sparse systems [1],[2],[3],[4],[5],[6],[7],[8],[9], [10],[18],[22],[23]. Mainly, two techniques are used to develop algorithms for sparse systems, one is the compressed sensing approach and the other is proportionate update scheme [2]. Many algorithms were inspired by compressive sensing. Different $\boldsymbol{l}$-norm concepts were applied to least mean square (LMS), recursive least square (RLS), affine projection (AP), and Maximum Correntropy Criterion (MCC) cost functions [3],[4],[5],[18], [21],[24],[25]. By vanishing the inactive weights, zero attractor (ZA) based adaptive filters improve the performance gain of sparse systems over their conventional counterparts especially in steady-state [4],[20].

The proportionate updating (PU) mechanism was popular for the systems having relatively nonuniform sparse structures [2]. The proportionate updating concept is applied to LMS family of algorithms. That improves the steady-state performance and convergence rate of the algorithm. For each filter coefficient, proportionate NLMS (PNLMS) has a different step size for individual coefficient. A proportionate matrix is introduced into the cost function to incorporate the step size for each individual coefficient, which is linearly proportional to the magnitude of that filter coefficient's estimation [6]. For sparse impulse response, PNLMS algorithm converges fast during the initial phase but afterward, convergence reduces significantly. For the dispersive impulse response of the system, convergence rate deteriorates even lower than NLMS algorithm. An improved PNLMS (IPNLMS) was introduced to address this issue, which exploits the proportionate concept by mixing NLMS and PNLMS algorithms in a controlled manner to handle proportionate and non-proportionate adaption [7]. Sparseness variation in impulse response changes the performance of IPNLMS. Sparseness controlled IPNLMS introduces a factor to control degree of sparsity into diagonal components of the proportional matrix. It increases the robustness of IPNLMS. [5],[8],[9].

Despite extensive research in the area of Proportionate Update (PU) in the context of LMS, AP, and MCC, efficient design of sparse RLS using this mechanism is still lacking [10],[11], [18]. Natural RLS (NRLS), proportionate RLS algorithms, L0-PRLS and L1-PRLS design using PU mechanism [8],[10],[11],[12]. A proportionate matrix was used to take into account the system sparsity in the natural recursive least square (NRMS). NRLS is more dependent on the conditions of the input covariance matrix than classical RLS [8]. By combining proportionate matrix with Kalman gain vector in traditional RLS update equation, a new algorithm proportionate RLS (PRLS) was proposed by yu wang and zhen qin in 2021 [23]. The PRLS was inspired by the concept of IPLMS [13],[14]. L0-PRLS and L1-PRLS were designed using and norm penalty in the cost function of RLS and a proportionate matrix in the update equation [11],[12],[20],[22],[23].

To enhance the performance of PRLS, a new algorithm is proposed by inserting sparseness control factor in the proportionate matrix. Performance of RLS, PRLS, and proposed Sparseness controlled proportionate recursive least square (SC-PRLS) is a compared for sparse system identification. This study also studies the effect of the convergence control parameter on the algorithm's performance.

The paper is organized as follows: in section 2, a brief description of different adaptive algorithms incorporating proportionate update (PU) mechanisms is presented. a new algorithm SC-PRLS is proposed in section 3. The simulation results and comparison are given in section 4. Section 5 concludes the paper.

2. RELATED ADAPTIVE ALGORITHMS FOR SSPARSE SYSTEMS

In this section, a brief introduction of the algorithms incorporating PU mechanism in LMS and RLS is presented, which inspired the development proposed algorithm. The LMS algorithm and its variants are widely used for linear filtering [3],[4],[8],[10],[11],[12],[13]. For non-stationary environment Normalised Least Mean Square (NLMS) uses adaptive step size.

The update equation for basic LMS algorithm is given as:

$$\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k+1) = \widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k) + \mu e(k)\widehat{\boldsymbol{x}}(k) \quad (1)$$

Where $e(k) = d(k) - \widehat{\boldsymbol{x}}^T(k)\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)$, is the instantaneous error, $d(k)$ is the desired signal, μ is the step size, $\widehat{\boldsymbol{x}}(k) = [x(k), x(k-1), \ldots\ldots, x(k-N+1)]^T$ is the input signal, and $\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k) = [w_1(k), w_2(k), \ldots\ldots, w_N(k)]^T$ is for adaptive filter's coefficient/tap vector of length N.

2.1 Normalized Least Mean Square (NLMS)

LMS has fixed step size. Proper selection of step size is important for convergence. When working environment becomes Non-stationary, LMS does not perform well. Therefor adaptive step size was introduced in NLMS. Which allows the filter adaption self-regulating and does not depends on incoming signal. The update equation for NLMS is given as:

$$\widehat{w}_N(k+1) = \widehat{w}_N(k) + \mu(k)\hat{x}(k)e(k) = \widehat{w}_N(k) + \mu_0 \frac{\hat{x}(k)e(k)}{\hat{x}^T(k)\hat{x}(k)+\delta_{NLMS}} \quad (2)$$

Where Adaptive step size $\mu(k) = \frac{\mu_0}{\|\hat{x}(k)\|_2^2+\delta_{NLMS}}$, μ_0 is initial step size, $\|.\|_2$ is the Euclidean norm and $\delta_{NLMS}$is the regularization parameter used to avoid the problem excessively high step size when magnitude of the input vectors is zero or near to zero.

2.2 Proportionate NLMS Algorithm (PNLMS)

LMS and NLMS algorithms do not work well when the desired impulse response has a high proportion of inactive weights compared to active weights. Therefore, Proportionate NLMS (PNLMS) was developed incorporating proportionate matrix in NLMS for the applications such as echo cancellation, adaptive noise cancellation, under water acoustic channel estimation etc. where the environment is sparce. In PNLMS, larger coefficients maintain large step size to make faster convergence. Therefore, active taps are adjusted faster than the non-active taps for the sparse system. It leads to faster convergence of PNLMS over NLMS for sparse systems. The filter update equation for the PNLMS algorithm is given as:

$$\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k) = \widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k-1) + \mu(\boldsymbol{G}(k-1)\widehat{\boldsymbol{x}}(k)e(k)) / (\widehat{\boldsymbol{x}}^T(k)\boldsymbol{G}(k-1)\widehat{\boldsymbol{x}}(k) + \delta_{PNLMS}) \quad (3)$$

The diagonal matrix **G**(n) leads to adjustment in the step-sizes of the individual weights of the filter. **G**(n) is known as proportionate matrix.

$$\boldsymbol{G}(k-1) = diag\{g_1(k-1), g_2(k-1), g_3(k-1), \ldots\ldots, g_N(k-1)\} \quad (4)$$

The diagonal elements of proportionate matrix G(n) are calculated as:

$$g_l(k) = \frac{\gamma_l(k)}{\frac{1}{N}\sum_{i=0}^{N-1}\gamma_i(k)}, 0 \le l \le N-1 \quad (5)$$

$$\gamma_l(k) = max\{\rho, \max(\delta_p, |w_0(k)|, |w_1(k)|, \ldots\ldots |w_{N-1}(k)|), |\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)|\} \quad (6)$$

Where $\delta_p$is regularization parameter and it is used to avoid $\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)$ from stalling during initialization stage. The parameter ρ in equation (6) is used to avoid the coefficients from inhibiting when they are relatively smaller than the largest one. Its typical value is from 1/*N* to 5/*N*. The regularization parameter for PNLMS is related with the regularization parameter of NLMS as $\delta_p = \frac{\delta_{NLMS}}{N}$, where N is length of filter.

The PNLMS algorithm has faster initial convergence and tracking than NLMS for sparse Impulse response. The main disadvantage of PNLMS algorithm is that after initial fast adaptation, its convergence rate begins to slow significantly and becomes even poor than the NLMS algorithm. For dispersive impulse response, convergence speed is less in PNLMS algorithm. The computational complexity is increased and slows the convergence speed after initial period.

2.3 Improved PNLMS (IPNLMS) Algorithm

To overcome the problem of slow convergence of PNLMS, several improved versions of PNLMS like IPNLMS and PNLMS++ are proposed. The PNLMS++ improves the convergence by alternating between proportionate and nonproportionate NLMS. A regulated combination of PNLMS and NLMS is used in the IPNLMS to prevent performance loss for dispersive impulse response of the unknown system. But for the initial phase, the IPNLMS converges similar to the PNLMS for sparse impulse response. So, overall performance of IPNLMS is better than PNLMS.

For IPNLMS, the diagonal elements of the proportional matrix G(n), which is a diagonal matrix, are defined as

$$\gamma_l(k) = (1-\alpha)\frac{\|\widehat{\boldsymbol{w}}(k)\|_1}{N} + (1+\alpha)|\widehat{\boldsymbol{w}}(k)| \quad (7)$$

$$g_l(k) = \frac{(1-\alpha)}{2N} + \frac{(1+\alpha)|w_l(k)|}{2\sum_{i=0}^{N-1}|w_i(k)|+\varepsilon}, 0 \le l \le N-1$$

$$g_l(k) = \frac{(1-\alpha)}{2N} + \frac{(1+\alpha)|w_l(k)|}{2\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_1+\varepsilon}, 0 \le l \le N-1 \quad (8)$$

Where $\|\cdot\|_1$ is define as the ℓ_1-norm. To prevent division by zero, a constant of very small positive value ε is incorporated. The adjustable parameter α can alter the behavior of IPNLMS by switching between NLMS and PNLMS. For α = -1, IPNLMS is equivalent to NLMS and IPNLMS behaves like PNLMS when α is close to 1.

The movable parameter can alter how IPNLMS behaves when switching between NLMS and PNLMS. As can be shown, IPNLMS behaves like PNLMS when = -1 and is identical to NLMS when = -1.

The taps update equation for IPNLMS is described as:

$$\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k) = \widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k-1) + \mu(\boldsymbol{G}(k-1)\widehat{\boldsymbol{x}}(k)e(k)) / (\widehat{\boldsymbol{x}}^T(k)\boldsymbol{G}(k-1)\widehat{\boldsymbol{x}}(k) + \delta_{IPNLMS}) \quad (9)$$

Where the regularization parameter $\delta_{IPNLMS} = \frac{(1-\alpha)\delta_{NLMS}}{2N}$.

Normalized misalignment is low in IPNLMS in comparison to PNLMS. For an impulse response that is between sparse and dispersive, it behaves much better than PNLMS and PNLMS++. Also, for highly sparse impulse response, IPNLMS algorithm converges same as PNLMS [6], [15],[16].

The IPNLMS improves the tracking capability for sparse impulse response and non-sparse or dispersive impulse response for time-varying system. when ε is decreased, the steady-state MSE increased.

2.4 Sparseness-controlled IPNLMS (SC-IPNLMS) Algorithm

The sparseness-controlled IPNLMS (SC-IPNLLMS) algorithms measure the sparseness of the predicted impulse response at each iteration, which improve the convergence rate for the systems with variable sparseness.

For impulse response $\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)$, the degree of sparseness is defines as [3],[17], [26]:

$$\xi(k) = \frac{N}{N-\sqrt{N}}\left\{1 - \frac{\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_1}{\sqrt{N}\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_2}\right\}$$

Where $0 \le \xi(k) \le 1$ and $\|.\|_1$ & $\|.\|_2$ are define as the ℓ_1-norm and ℓ_2-norm respectively. Impulse responses that are dispersive or non-sparse have tiny values of ξ(k) that are close to 0, while impulse responses that are sparse have higher values of ξ(k) that is close to 1.

The convergence of SC-IPNLMS of is divide in 2 stages. In 1st stage it follows IPNLMS and in 2nd stage it uses SC-IPNLMS that uses degree of sparseness [9].

The control factor of SC-IPNLMS algorithm is:

$$g_l(k) = \frac{(1-\alpha)}{2N} + \frac{(1+\alpha)|w_l(k)|}{2\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_1+\delta_{IPNLMS}}, \text{for } 0 \le k \le N-1 \quad (11a)$$

$$g_l(k) = \left[\frac{1-0.5\xi(k)}{N}\right]\frac{(1-\alpha_{SC-IPNLMS})}{2N} + \left[\frac{1+0.5\xi(k)}{N}\right]\frac{(1+\alpha_{SC-IPNLMS})|w_l(k)|}{2\|\widehat{\boldsymbol{w}}_N(k)\|_1+\delta_{SC-IPNLMS}}, for\ k \geq N \quad (11b)$$

To improve convergence, SC-IPNLMS adjust step size of individual filter taps by measuring the degree of sparseness for estimated impulse response.

2.5 Recursive least square algorithm (RLS)

The filter weights in RLS can be updated as

$$\widehat{\boldsymbol{w}}_N(k) = \widehat{\boldsymbol{w}}_N(k-1) + \boldsymbol{\mathcal{K}}(k)e^*(k|k-1) \quad (12)$$

$$e(k|k-1) = d(k) - \widehat{\boldsymbol{w}}_N{}^H(k-1)\widehat{\boldsymbol{x}}(k) \quad (13)$$

$$\boldsymbol{\mathcal{K}}(k) = \frac{\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)}{\lambda+\widehat{\boldsymbol{x}}^H(k)\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)} \quad (14)$$

forgetting factor $0 < \lambda < 1$

$$\boldsymbol{P}(k) = \lambda^{-1}[\boldsymbol{P}(k-1) - \boldsymbol{\mathcal{K}}(k)\widehat{\boldsymbol{x}}^H(k)\boldsymbol{P}(k-1)] \quad (15)$$

2.6 proportionate recursive least square (PRLS) algorithms

Using the idea of IPNLMS, the PRLS algorithm is created in [10]. Inspired by IPNLMS, a Proportionate matrix G(k-1) is added to the RLS update equation. The diagonal matrix G(k-1) was produced via equation given below

$$\boldsymbol{G}(k-1) = diag\{g_1(k-1), g_2(k-1), \dots\dots, g_N(k-1)\} \quad (16)$$

$$g_n(k-1) = \frac{\mu(1-\alpha)}{2N} + \mu(1+\alpha)\frac{|w_n(k-1)|}{2\|\widehat{\boldsymbol{w}}_N(k-1)\|_1+\epsilon}, \quad for\ n = 1,2,3\dots.N \quad (17)$$

Where $\alpha \in [-1, 1)$ and ϵ is regularized parameter.

Filter weight update equation is

$$\widehat{\boldsymbol{w}}_N(k) = \widehat{\boldsymbol{w}}_N(k-1) + \boldsymbol{G}(k-1)\boldsymbol{\mathcal{K}}(k)e^*(k|k-1) \quad (18)$$

Where A priori error

$$e(k|k-1) = d(k) - \widehat{\boldsymbol{w}}_N{}^H(k-1)\widehat{\boldsymbol{x}}(k) \quad (19)$$

Where $\boldsymbol{\mathcal{K}}(k)$ Kalman gain vector

$$\boldsymbol{\mathcal{K}}(k) = \frac{\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)}{\lambda+\widehat{\boldsymbol{x}}^H(k)\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)} \quad (20)$$

The inverse of input covariance matrix computed similar to RLS.

The principle of the PRLS is to use the proportionate matrix G(n-1) to determine the large weights to be assigned to active taps in a sparse environment. A control parameter μ in the proportionate matrix used for balancing PRLS's convergence performance and steady-state behaviour.

3. ORIGINALITY

A novel adaptive digital signal processing technique for sparse system identification is presented in this research. The algorithms designed for sparse systems does not perform well for non-sparse systems. It performs well for sparse as well as non-sparse system. When Degree of sparse changes performance of proportionate type algorithms do not perform well, while the proposed algorithm has consistent performance even when degree of sparseness changes.

4. PROPOSED SPARSENESS-CONTROLLED PROPORTIONATE RECURSIVE LEAST SQUARE (SC-PRLS) ALGORITHM

A new algorithm is proposed in this section which is inspired from SC-IPNLMS and PRLS. The sparseness value of the estimated impulse response at each iteration serves as the basis for the algorithm. Proportionate matrix control parameters are computed based on the sparseness level Accordingly, it controls the weights of each tap during the update process. The proposed sparseness-controlled algorithm improves the convergence rate as compared to PRLS for both sparse and non-sparse systems. In SC-PRLS, as the estimate of the filter coefficients $\widehat{\boldsymbol{w}}_N(k)$ gradually converges, the sparseness measurement converges to its optimal value, which is the sparseness of actual impulse response of unknown system. The sparseness of an impulse-responsive system is measured by its degree of sparseness ξ(k). $\xi(k)$ is defined in equation number (10). When the degree of sparseness is one, only one tap is nonzero and rest all taps are zero. For the degree of sparseness is zero, all taps are equal including sign. So, degree of sparseness depends on number of nonzero coefficients in estimated impulse response. Degree of sparseness $\xi(k)$ is expected to converges faster than $\widehat{\boldsymbol{w}}_N(k)$ as it is more sensitive to the fluctuations of $\widehat{\boldsymbol{w}}_N(k)$ around its optimal value.

Proportionate update (PU) leads to gain distribution in accordance with diagonal elements of the proportionate matrix. So, the sparseness control is included in the elements of proportionate matrix, similar to the implementation in SC-IPNLMS [9]. The proportionate control distribution parameters $g_l(k)$ for the proposed algorithms can be computed by equation number (21), in which the proportionate and nonproportionate terms are combined.

The control factors of SC-PRLS algorithm are computed as follows:

$$g_l(k) = \mu\left[\frac{a(k)}{N}\right]\frac{(1-\alpha_{sc})}{2N} + \mu\left[\frac{b(k)}{N}\right]\frac{(1+\alpha_{sc})|w_l(k)|}{\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_1+\delta}, \quad (21)$$

Where $\alpha_{sc} \in [-1,1]$ is a constant, $a(k) = 1 - \theta\xi(k)$ and $b(k) = 1 + \theta\xi(k)$ enables the negative and positive changes as degree of sparseness $\xi(k)$ increases. The range of $a(k)$ and $b(k)$ is $1 < a(k) < 1-\theta$ and $1 < b(k) < 1+\theta$ corresponding to contributions of non-sparse and sparse terms in the elements of proportionate control matrix. To further enhance performance, weights are assigned based on a priori knowledge of spareness using the constant θ. Non-proportional terms are given an excessive amount of weight when the degree of sparseness is not considerable, while proportionate terms become less significant. Typically, unknown sparse systems have θ = 0.5 [26].

$$g_l(k) = \mu\left[\frac{1-\theta\xi(k)}{N}\right]\frac{(1-\alpha_{sc})}{2N} + \mu\left[\frac{1+\theta\xi(k)}{N}\right]\frac{(1+\alpha_{sc})|w_l(k)|}{\|\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k)\|_1+\delta} \quad (22)$$

For large value of $\xi(k)$, the impulse response is sparse and algorithm allocates more weight to the proportionate term, for less sparse impulse response, the algorithm allocates higher weightage to non-sparse term. Second term in the equation number (24) is responsible for individual weight updates, while first term is equally applied to all the weights. Both terms lead to faster convergence when impulse response is relatively less sparse. To speed up the convergence of large filter coefficients, the initial value of degree of sparseness ξ (0) should be high. The convergence controlling parameter μ is responsible for trade off the between steady-state behaviour and convergence rate.

A Proportionate matrix $\boldsymbol{G}(k-1)$ is a diagonal matrix whose diagonal elements are calculated by equation number (24). The matrix $\boldsymbol{G}(k-1)$ is defined as

$$\boldsymbol{G}(k-1) = diag\{g_1(k-1), g_2(k-1), \ldots\ldots, g_N(k-1)\} \quad (23)$$

This proportionate matrix $\boldsymbol{G}(k-1)$ introduced in filter update equation of SC-PRLS. The filter weight update equation for the proposed algorithm is given as

$$\widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k) = \widehat{\boldsymbol{w}}_{\boldsymbol{N}}(k-1) + \boldsymbol{G}(k-1)\boldsymbol{\mathcal{K}}(k)e^*(k|k-1) \quad 24)$$

Where, a priori error is defined as

$$e(k|k-1) = d(k) - \widehat{\boldsymbol{w}}_{\boldsymbol{N}}{}^H(k-1)\widehat{\boldsymbol{x}}(k) \quad (25)$$

The Kalman gain vector in the tap update equation is define as

$$\boldsymbol{\mathcal{K}}(k) = \frac{\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)}{\lambda+\widehat{\boldsymbol{x}}^H(k)\boldsymbol{P}(k-1)\widehat{\boldsymbol{x}}(k)} \quad (26)$$

Inverse of input covariance matrix P(n-1) is defined similar to the standard RLS.

The proposed algorithm uses sparseness to update active and passive taps separately. Depending the on degree of sparseness it assigns the weight to proportionate and nonproportionate term. Proportionate matrix is updated after every iteration in update equation and all other parameters are calculated similar to standard RLS.

5. RESULTS AND ANLYSIS

Simulations have been performed using MATLAB and performance of PRLS, RLS and proposed SC-PRLS algorithms is compared. Mean squared error (MSE) and Means Square Deviation (MSD) is calculated by applying independent random input signal of length N=2000 samples. MSE and MSD is averaged after each iteration for 2000 trials. Forgetting factor was kept 0.99, $\alpha_{sc} = 0.65$ regularization parameter $\delta = 0.001$ and convergence controlling parameter μ=35. Fig.1 shows mean square error and steady-state error performance of the algorithm. Here, the number of active taps was kept at 10% of the total taps. Total number of taps was kept at 40. The variations of mean square deviation with respect to number of iterations is shown in fig.2.

The convergence of the algorithms takes place at 230 iterations, 380 iteration and 800 iterations for SC-PRLS, PRLS and RLS respectively. steady-state error (SSE) is -40dB, -43db, -43.2db for PRLS, RLS & SC-PRLS respectively. From the results, it can be observed that RLS has better steady-state error performance but converges slowly while PRLS converges with the improved rate of convergence but steady-state error performance is poor than RLS. While SC-PRLS has better convergence rate, steady-state error performance and MSD in comparison to the RLS and PRLS algorithms.

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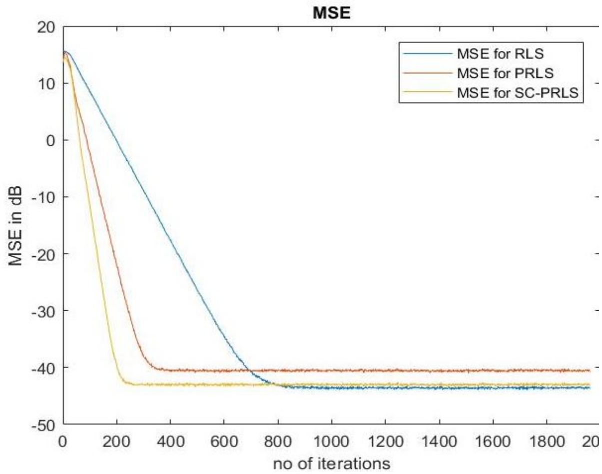


Fig.1 Comparison of MSE for RLS, PRLS and SC-PRLS

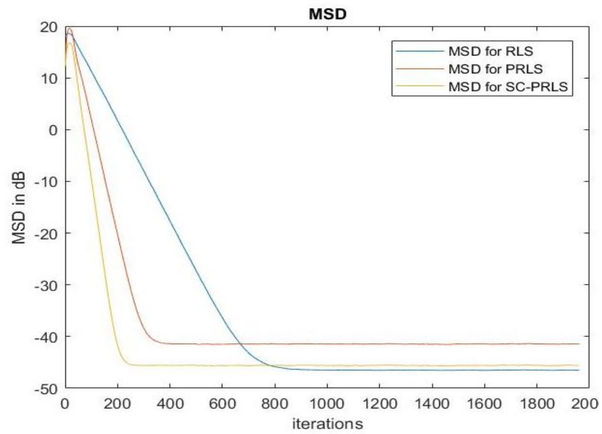


Fig.2 Comparison of MSD for RLS, PRLS and SC-PRLS.

To analyze SC-PRLS for different sparse conditions like sparse, relatively less sparse and non-sparse, impulse responses were chosen accordingly. For sparse system, only 4 taps are active out of total 60 taps, these 4 taps assigned the value one. The input is random values of 0 or 1. For relatively less sparse system, 20 active taps are assigned different values in the range of 1 to 9. For non-sparse systems, all taps are nonzero and input is randomly generated.

The performance of SC-PRLS for different sparse values is shown in in fig.3. The algorithm has consistency in rate of convergence but steady-state error increases from sparse to non-sparse systems. Mean square error converges near 230 iterations for all but steady-state error for sparse environment is nearly -59 dB, for relatively sparse -38dB, whereas non-sparse has -25 dB of steady-state error. SC-PRLS performs better in both MSE and SSE for sparse systems and then steady-state error increases as system characteristics changes from sparse to non-sparse. In non-sparse environment also, the proposed algorithms have satisfactory performance.

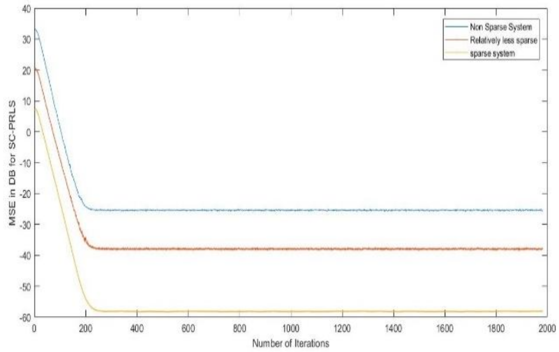


Fig.3 Mean Square Error variation of SC-PRLS for sparse, relatively less sparse and non-sparse system.

When number of filter weights increases considerably, convergence rate decreases, however by decreasing μ, convergence rate can be improved without requirement of large input samples. If μ value is not chosen properly then the algorithm may also diverge instead of converging. Therefore, performance of SC-PRLS also depends on convergence control parameter μ and its effect on MSE is shown in fig.5 for fixed tap size and different values of μ. The convergence control parameter μ has a range for better performance of the algorithm.

The rate of convergence decreases beyond this range and afterword algorithm stops converging. Observation of fig.4 indicates that the range of μ from 30-85 leads to better performance and below $\mu = 30$ rate convergence decreases but steady-state error improves and keep on improving at the cost of convergence. After $\mu = 0.5$ algorithm diverges instead of converging. If μ is increased beyond 85 then convergence as well as steady-state error both deteriorate and after $\mu = 122$, algorithm stops converging. Such variation is visible in fig.4, fig.5 and fig.6. The graph of fig.5 shows the variation in steady-state error v/s μ for the proposed algorithm.

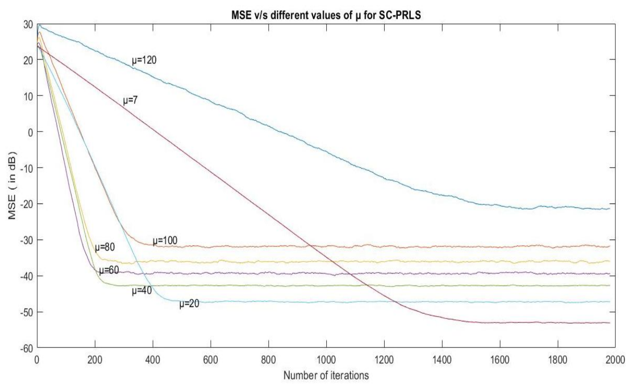
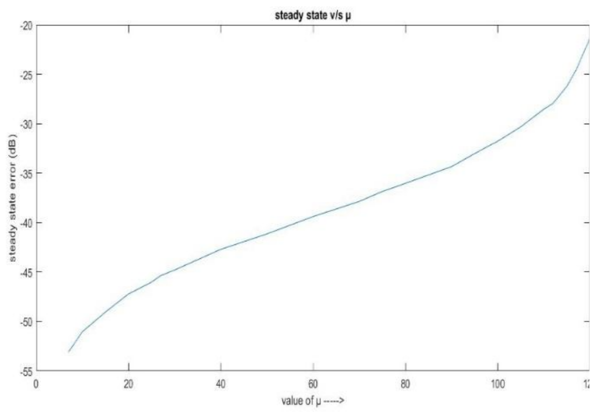


Fig.4 MSE for SC-PRLS with fixed tap size for different values of μ



Fg.5 Steady-state error verses μ for SC-PRLS

Change in convergence rate for different μ is clearly visible in Fig.6 and it helps to determine the optimum range for μ for which convergence rate is better.

It was also observed that the range of μ decrease as the number of filter taps increase. During the experiment, number of filter weights were varied from 10 to 500. Better convergence rate is observed for the values of μ in the range of 80 to 20. When number of filter weights are 500, the algorithms diverge for μ =50 while the number of filter weights are 20 then the algorithms show better convergence for same value of μ. So, there is trade-off between number of filter weights and rate of convergence. The wrong selection of μ leads to degradation of performance. The algorithm may diverge if μ is beyond upper bound.

Proposed SC-PRLS compared to RLS and PRLS by varying the value of constant α_{sc} and convergence controlling parameter μ. For $\alpha_{sc} = 0.925$, the SC-PRLS performs similar to RLS and for $\alpha_{sc} = 0.4$, it performs similar to PRLS. Further decrease in α_{sc} , deteriorates the performance of the algorithm. optimum value α_{sc} is 0.65 for better performance of SC-PRLS.

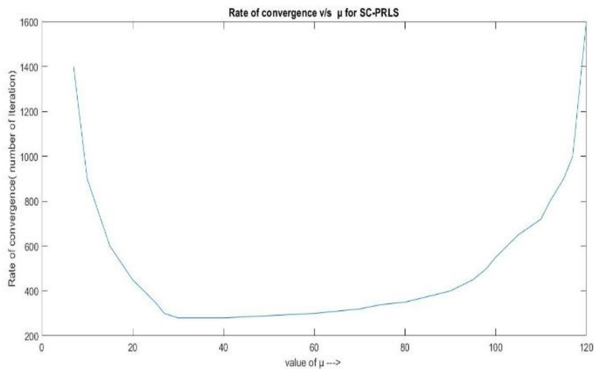


Fig.6 Rate of convergence when μ is changes for SC-PRLS

The SC-PRLS performs similar to PRLS at $\mu = 75$ and for $\mu > 75$, PRLS is better than SC-PRLS, while SC-PRLS perform better than PRLS for $28 < \mu < 75$. If μ is further decreases then steady-state error is better than PRLS but rate of convergence is not better than PRLS. The optimum value of μ for SC-PRLS is 35 where MSE and steady-state error both are better than PRLS.

Compared to RLS and PRLS, the proposed SC-PRLS algorithm performs better in terms of steady-state error performance and convergence rate.

6. Conclusion

The SC-PRLS algorithm is proposed by incorporating the degree of sparseness into the diagonal elements of the proportionate matrix of PRLS. This algorithm incorporates convergence control factor that can be adjusted to improve the steady-state error and rate of convergence whenever the number of weights is significant. Proposed SC-PRLS algorithm perform well in both sparse and non-sparse systems. As the degree of sparseness changes, the effect of the proportionate term and non-proportionate term in the algorithm varies accordingly. The simulation results indicate improved convergence of SC-PRLS over RLS and PRLS. Also, it has a better steady-state error. The steady-state error improves when the degree of sparseness increases. The simulation results also indicate the dependence of steady-state error and convergence on the control parameter. By choosing an appropriate value of the control parameter both can be improved.

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