

G-Jitter Effects on Transient Natural Convection Couette Flow in a Vertical Channel

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ABSTRACT

This article investigates the effects of g-jitter on transient natural convection Couette flow in a vertical channel. Fully developed laminar time dependent flow is considered. The flow formation is caused by the buoyancy force arising from the temperature gradient as a result of asymmetric heating of parallel plates as well as constant motion of one of the plates. The method of Laplace transforms is used to obtain the expressions for temperature and velocity which are used to compute the Nusselt number, skin-friction, mass flux and mean temperature in Laplace domain then the Riemann-sum approximation is used to invert our expressions from Laplace domain to time domain. During the course of investigation, it was found that the influence of heat generation/absorption parameter on the rate of heat transfer on one plate is the reverse of the influence on the other plate. As the amplitude of the g-jitter increases, the velocity of the fluid increases and also the influence of frequency of oscillations of g-jitter on mass flux and mean temperature depends on time.

Keywords - Vertical channel, Natural convection, Couette flow, g-jitter.

Date of Submission: January 4, 2023

Date of Acceptance: February 20, 2023

1 INTRODUCTION

One of the basic flows in fluid mechanics is the Couette flow where the fluid motion is induced by movement of the bounding surface. The flow of a viscous incompressible fluid past an impulsively started horizontal flat plate was first studied by Stokes [1]. Singh and Kumar [2] investigated the formation of Couette flow of a viscous, incompressible and electrically conducting fluid between two infinite parallel plane walls in the presence of transverse magnetic field, by assuming that the magnetic lines of force are fixed relative to the moving plate for both the impulsive and accelerated motion of the moving plates and came out with the deduction that magnetic field increases the velocity field in both cases. The effect of natural convection on unsteady Couette flow was studied by Singh [3]. The Laplace transform technique was used to obtain the velocity and temperature fields, the skin-friction and rate of heat transfer. It was observed that an increase in the Grashof number results in an increase in the flow velocity. Jha[4] extended the work of Singh [3] by discussing the combined effects of natural convection and a uniform transverse magnetic field when the magnetic field is fixed relative to the plate or fluid. Using the Laplace transform technique, exact solutions were obtained for the velocity and temperature fields. The trends observed with respect to the magnetic parameter were consistent with those observed in Rossow[5]. Singh et al. [6] examined the effect of rotation on the unsteady hydromagnetic Couette flow when one of the plates has been set into accelerated motion; they noticed a decrease in velocity due to rotation and an increase due to magnetic field. Fully-developed laminar free convection Couette

flow between two vertical parallel plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion has been analyzed by Jain and Gupta [7]. The physical effect of external shear in the form of Couette flow of a Bingham fluid in a vertical parallel plane channel with constant temperature differential across the walls was investigated analytically by Barletta and Magyari[8]. Steady fully-developed combined forced and free convection Couette flow with viscous dissipation in a vertical channel has been investigated analytically by Barletta *et al.* [9]. In this study, the moving wall is thermally insulated and the wall at rest is kept at a uniform temperature. The natural convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation has been studied by Narahari[10].

The study of heat generation/absorption effects in moving fluids is of importance in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids, etc. Among the foundational works on effect of periodic heating on flows are the works of Sparrow and Gregg [11] and Chung and Anderson [12] that considered cases in which the surface temperature varies slightly about a Mean level, which is higher than the ambient temperature. Their studies were restricted to small amplitudes of the surface temperature variation. Nanda and Sharma [13] were also able to circumvent the limitation of the result for only small amplitude by separating the temperature and velocity into steady and oscillatory components, by using

perturbation expansion; their results were restricted to small amplitudes. Foraboschi and Federico [14] presented the volumetric rate of heat generation/absorption which is directly proportional to $(T - T_0)$ and explained that it is an approximation of the state of some exothermic process with T_0 as the initial temperature. Jha [15] conducted a study on flow behaviour of a transient free convective flow in a vertical channel due to asymmetric heating of the channel walls in the presence of temperature-dependent heat generating/absorbing. Free convective flow of heat generation/absorption fluid between vertical porous plates with periodic heat input has been studied by Jha and Ajibade [16]. Also, Jha *et al.* [17] considered the effect of heat generation/absorption on natural convection flow near an infinite vertical plate with ramped temperature. Umavathi and Sultana [18] studied the effect of mixed convection in a vertical channel with porous medium in the presence of heat generation/absorption parameter. It was found that as the heat absorption parameter increases the velocity decreases, but the Nusselt number is an increasing function of the heat generation coefficient.

Recently, there has been a great deal of interest in the engineering study of the effects of complex body forces on fluid motion. Such forces can arise in a number of ways, for example, when a system with density gradients is subject to vibrations. The resulting buoyancy forces, which are produced by the interaction of density gradients and the spatial and frequency distribution of the vibration-induced acceleration field on fluid motion is known as gravity modulation or g-jitter induced flow. G-jitter is defined as the inertia effect due to quasi-steady, oscillatory or transient acceleration arising from aircraft's crew motions and machinery vibrations in parabolic aircrafts, space shuttles or other microgravity environments [22]. Several attempts have been made to estimate and calculate the effects of time varying g-jitter on the convective flow. Amin [23] investigated the heat transfer from a sphere immersed in an infinite viscous and incompressible fluid in a zero gravity environment under the influence of g-jitter. She has shown that heat transfer is negligibly small for high-frequency g-jitter but under special circumstances, when the Prandtl number is sufficiently high, low-frequency g-jitter may play an important role. Chamkha [24] has studied the oscillating free and mixed convection flow driven by g-jitter forces associated with microgravity and the magnetic field effect for a system consisting of two parallel plates heated at different temperatures. Sharidan *et al.* [25] has investigated the effects of g-jitter induced and combined with heat and mass transfer by mixed convection

flow in microgravity situation for a simple system. This system consists of two heated vertical parallel infinite flat Effects of G-jitter Combined with Heat and Mass Transfer by Mixed Convection flow in microgravity situation for a simple system. This system consists of two heated vertical parallel infinite

flat plate held at constant but different temperatures and concentrations for Newtonian fluid. However, to the best of our knowledge, there are no current attempts being made for the effect of g-jitter on transient natural convection Couette flow in a vertical channel.

Jha and Ajibade [19] worked on an exact analysis of unsteady free convection Couette motion between two vertical parallel plates, where the moving plate is subjected to constant heating and the plate at rest is isothermally cold. In their work, the acceleration due to gravity is considered to be constant. However, the value of acceleration due to gravity fluctuates about a Mean value (9.79787-9.81792) (Nethercott and Walton [20]), so that the result obtained by Jha and Ajibade [19] cannot adequately capture the real life situation. Therefore it is desirable to address the shortcomings of Jha and Ajibade [19], this work remodels their work considering a fluctuating acceleration due to gravity (g-jitter).

The aim of the present paper is to provide an analysis of g-jitter effect on transient natural convection Couette flow in a vertical channel where the moving plate is subjected to constant heating and the plate at rest is isothermally cold. The method of Laplace transforms is used to obtain the expressions for temperature and velocity which are used to compute the Nusselt number, skin-friction, mass flux and mean temperature and later Riemann-sum approximation is used to invert our expression from Laplace domain to time domain due to difficulty in the direct inversion. Considering g-jitter effects in natural convection flow is expected to give a degree of freedom to the different flow phenomena in the system and as such, designers of engineering systems can use this freedom in improving their designs.

2 MATHEMATICAL ANALYSIS

Transient natural Convection flow of a viscous incompressible heat generating/absorbing fluid is considered in a vertical channel bounded by two infinite parallel plates. The flow is assumed to be in x' -direction which is taken vertically along one of the plates while y' -axis is taken normal to it. The second plate is placed h distance away from the first. At time $t' \leq 0$, the fluid is at rest, the temperature of fluid and that of the channel plates are kept at T_0 . At time $t' > 0$, the temperature of the plates $y' = 0$ raised or fell to T_w and thereafter maintained constant while the other plate $y' = h$ remains at T_0 . Also, the plate $y' = 0$ moves in its own plane impulsively at a uniform velocity $u' = U$ while the other plate remains at rest, the acceleration due to gravity is assumed to fluctuate sinusoidally with t , amplitude ε and frequency ω . The flow configuration and coordinates system is shown in *Figure-1 below*

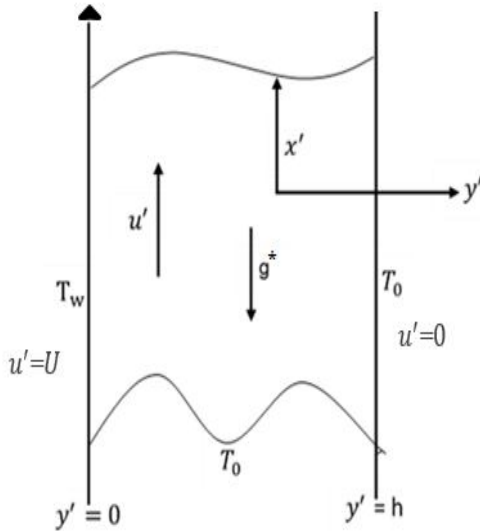


Figure1: Schematic diagram of the flow

Note that the g -jitter is fluctuating with amplitude ϵ and frequency ω and also as the amplitude approaches zero, the acceleration due to gravity becomes constant so that the problem coincides with the work of Jha and Ajibade [19].

Then under the usual Boussinesq's approximation the flow is shown to be governed by the following system of equations :

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g^* \beta (T' - T_0) \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0 (T' - T_0)}{\rho C_p} \quad (2)$$

Where acceleration due to gravity is given as $g^* = g(1 + \epsilon \sin \omega' t')$ Ramos [21].

with the initial condition

$$u'(0, y) = T'(0, y) = 0 \quad (3)$$

And the boundary conditions as: $t > 0$: $\begin{cases} u'(t, 0) = U & T'(t, 0) = T_w \\ u'(t, h) = 0 & T'(t, h) = T_0 \end{cases} \quad (4)$

In other to solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$y = \frac{y'}{h}, \quad t = \frac{t' \nu}{h^2}, \quad u = \frac{u'}{U}, \quad \theta = \frac{T' - T_0}{T_w - T_0}, \quad \omega = \frac{\omega' h^2}{\nu}$$

$$\delta = \frac{Q_0 h^2}{k}, \quad Pr = \frac{\mu C_p}{k}, \quad Gr = \frac{g \beta h^2 (T_w - T_0)}{\nu U}, \quad (5)$$

Pr is the Prandtl number which is inversely proportional to the thermal diffusivity of the working fluid, δ is the heat generation/absorption parameter, positive values denote absorption while negative values denote heat generation, Gr is the Grashof number. The physical quantities used in eq. (5) are defined in the nomenclature.

Upon substitution of equation (5) in to (1)-(4), the following equations are rendered in dimensionless form as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr \theta (1 + \epsilon \sin \omega t) \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{\delta}{Pr} \theta \quad (7)$$

$$u(0, y) = \theta(0, y) = 0 \quad (8)$$

With boundary conditions in dimensionless form as:

$$t > 0: \begin{cases} u(t, 0) = 1 & \theta(t, 0) = 1 \\ u(t, 1) = 0 & \theta(t, 1) = 0 \end{cases} \quad (9)$$

Introducing the Laplace transformations which are:

$$L(u(t, y)) = \tilde{u}(s, y) = \int_0^\infty u \exp(-st) dt \quad (10)$$

$$L(\theta(t, y)) = \tilde{\theta}(s, y) = \int_0^\infty \theta \exp(-st) dt \quad (11)$$

$$L(\theta(t, y) \exp(i\omega t)) = \tilde{\theta}(p, y) = \int_0^\infty \theta \exp(-pt) dt \quad (12)$$

$$L(\theta(t, y) \exp(-i\omega t)) = \tilde{\theta}(q, y) = \int_0^\infty \theta \exp(-qt) dt \quad (13)$$

(where s, p, q are the Laplace parameters such that $s > 0, p > 0, q > 0$) then applying the properties of Laplace transformation above on equation (6) and (7) gives:

$$\begin{aligned} \frac{d^2 \tilde{U}(s, y)}{dy^2} - s \tilde{u}(s, y) &= -Gr (\tilde{\theta}(s, y) + \frac{\epsilon}{2i} (\tilde{\theta}(p, y) - \tilde{\theta}(q, y))) \quad (14) \end{aligned}$$

$$A \frac{d^2 \tilde{\theta}(s, y)}{dy^2} - s \tilde{\theta}(s, y) = B \tilde{\theta}(s, y) \quad (15)$$

With boundary conditions in dimensionless form as:

Where A,B,p and q are defined in the Appendix

Boundary conditions as:

$$s > 0: \begin{cases} \vec{u}(s, 0) = \frac{1}{s} & \vec{\theta}(s, 0) = \frac{1}{s} \\ \vec{u}(s, 1) = 0 & \vec{\theta}(s, 1) = 0 \end{cases} \quad (16)$$

The solution of equation (14) and (15) under the boundary condition (16) in Laplace domain is given as:

$$\vec{\theta} = C_1 \exp(my) + C_2 \exp(-my) \quad (17)$$

$$\vec{u} = C_3 \exp(m^p y) + C_4 \exp(-m^p y) + a_0 \exp(my) + a_1 \exp(-my) + a_2 \exp(m_1 y) + a_3 \exp(-m_1 y) + a_4 \exp(m_2 y) + a_5 \exp(-m_2 y) \quad (18)$$

Where $C_3, C_4, a_0, a_1, a_2, a_3, a_4, a_5, C_1$ and C_2 are defined in the Appendix

Using the expressions (17) and (18), the rate of heat transfer from plate to fluid which is the Nusselt number, the shear stress on the boundary surface called the volumetric flow rate also known as skin-friction, mass flux and Mean temperature can be given in Laplace domain respectively as:

Nusselt number

$$\vec{Nu} = \frac{d\vec{\theta}}{dy} = mc_1 \exp(my) - mc_2 \exp(-my) \quad (19)$$

So that when $y = 0$ the Nusselt number becomes

$$\vec{Nu}_0 = \frac{d\vec{\theta}}{dy} \Big|_{y=0} = mc_1 - mc_2 \quad (20)$$

And also on the surface of the cold plate (i.e. when $y = 1$) the Nusselt number becomes

$$\vec{Nu}_1 = \frac{d\vec{\theta}}{dy} \Big|_{y=1} = mc_1 \exp(m) - mc_2 \exp(-m) \quad (21)$$

Skin-friction

$$\vec{\tau} = \frac{d\vec{u}}{dy} = C_3 m^p \exp(m^p y) - C_4 m^p \exp(-m^p y) + a_0 m \exp(my) - a_1 m \exp(-my) + a_2 m_1 \exp(m_1 y) - a_3 m_1 \exp(-m_1 y) +$$

$$a_4 m_2 \exp(m_2 y) - a_5 m_2 \exp(-m_2 y) \quad (22)$$

So that when $y = 0$ the skin-friction becomes

$$\vec{\tau}_0 = \frac{d\vec{u}}{dy} \Big|_{y=0} = C_3 m^p - C_4 m^p + a_0 m - a_1 m + a_2 m_1 - a_3 m_1 + a_4 m_2 - a_5 m_2 \quad (23)$$

And also on the surface of the cold plate (i.e. when $y = 1$) the Skin friction becomes

$$\vec{\tau}_1 = \frac{d\vec{u}}{dy} \Big|_{y=1} = C_3 m^p \exp(m^p) - C_4 m^p \exp(-m^p) + a_0 m \exp(m) - a_1 m \exp(-m) + a_2 m_1 \exp(m_1) - a_3 m_1 \exp(-m_1) + a_4 m_2 \exp(m_2) - a_5 m_2 \exp(-m_2) \quad (24)$$

Mass flux

The mass flux within the channel is also an important phenomenon that needs to be investigated. This is done by evaluating the sum total of fluid velocity over the entire flow domain

$$\vec{Q} = \int_0^1 \vec{u} dy = \frac{C_3}{m^p} (\exp(m^p) - 1) - \frac{C_4}{m^p} (\exp(-m^p) - 1) + \frac{a_0}{m} (\exp(m) - 1) - \frac{a_1}{m} (\exp(-m) - 1) + \frac{a_2}{m_1} (\exp(m_1) - 1) - \frac{a_3}{m_1} (\exp(-m_1) - 1) + \frac{a_4}{m_2} (\exp(m_2) - 1) - \frac{a_5}{m_2} (\exp(-m_2) - 1) \quad (25)$$

Mean temperature

The average temperature within the flow domain is given by the quotient of the bulk temperature and the mass flux

$$\vec{\theta}_m = \frac{\int_0^1 \vec{u} \vec{\theta} dy}{\int_0^1 \vec{u} dy} = \vec{\theta}_b = \frac{C_1 C_3}{m+m^p} (\exp(m + m^p) - 1) + \frac{C_1 C_4}{m-m^p} (\exp(m - m^p) - 1) +$$

$$\begin{aligned}
 & \frac{c_1 a_0}{2m} (\exp(2m) - 1) + a_1 C_1 + \\
 & \frac{c_1 a_2}{m_1+m} (\exp(m_1 + m) - 1) + \frac{c_1 a_3}{m-m_1} (\exp(m - \\
 & \quad m_1) - 1) + \frac{c_1 a_4}{m_2+m} (\exp(m_2 + m) - 1) + \\
 & \frac{c_1 a_5}{m-m_2} (\exp(m - m_2) - 1) + \\
 & \frac{c_2 C_3}{m^p-m} (\exp(m^p - m) - 1) - \\
 & \frac{c_2 C_4}{m+m^p} (\exp(-(m + m^p)) - 1) + a_0 C_2 - \\
 & \frac{c_2 a_1}{2m} (\exp(-2m) - 1) + \frac{a_2 C_2}{m_1-m} (\exp(m_1 - \\
 & \quad m) - 1) - \frac{a_3 C_2}{m_1+m} (\exp(-(m_1 + m)) - 1) + \\
 & \frac{a_4 C_2}{m_2-m} \exp(m_2 - m) - 1) - \\
 & \frac{c_2 a_5}{m_2+m} \exp(-(m_2 + m)) - 1) \quad (27)
 \end{aligned}$$

Since we know that $\vec{Q} = \int_0^1 \vec{u} dy$ in equation (25) then mean temperature can be represented in Laplace domain as

$$\vec{\theta}_m = \frac{\int_0^1 \vec{u} \vec{\theta} dy}{\int_0^1 \vec{u} dy} = \frac{\vec{\theta}_b}{\vec{Q}} \quad (28)$$

3 Results and Discussion

An unsteady free convective Couette flow of viscous incompressible fluid is considered in a vertical channel formed by two infinite vertical parallel plates. The plates are subjected to asymmetric thermal conditions and one of the plates moves impulsively with uniform velocity in its own plane. The objective of the present work is to investigate the influence of Grashof number (Gr), Prandtl number (Pr), g-jitter (g^*) and heat generation/absorption (δ) on fluid temperature, fluid velocity, rate of heat transfer, Nusselt number and skin-friction on the boundary plates. The values of the Prandtl number are taken to be 0.71 (prandtl number of air), 7.0 (Prandtl number of water), 2.0 (Prandtl number of sulphur dioxide), 3.0 (Prandtl number of liquid freon). The Grashof number which depends on the plate's thermal status takes positive, zero and negative values depending on the temperature difference between the plates, while in our work we arbitrarily take the values of the Grashof number to be 18, 45 and 89. The values of heat generation/absorption are taken in our work within the range of $(-4 \leq \delta \leq 4)$ and time also is taken within the range of $(0.1 \leq t \leq 0.7)$, the amplitude of the fluctuating acceleration due to gravity is taken to be less than unity ($\epsilon < 1$) and also the oscillating frequency (ω) is taken arbitrarily as $\frac{\pi}{18}, \frac{\pi}{3}, \frac{11\pi}{18}, \frac{17\pi}{18}$.

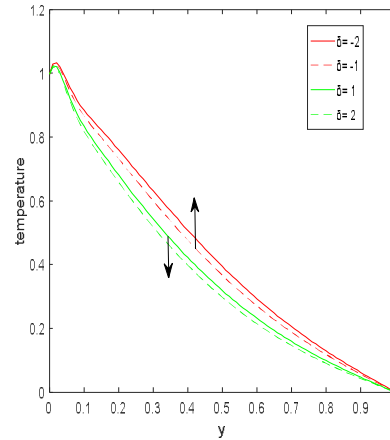


Figure2: Temperature profile for different heat generation/absorption (Pr=0.71, t=0.1)

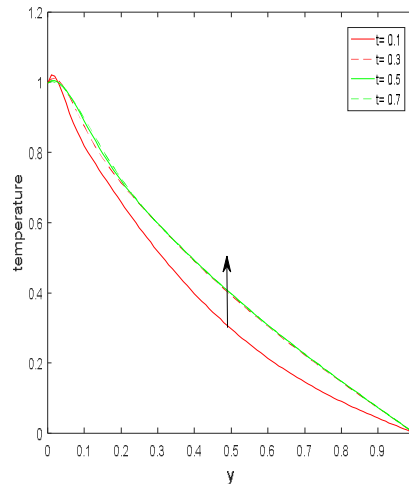


Figure3: Temperature difference for different time (Pr=0.71, $\delta=2$)

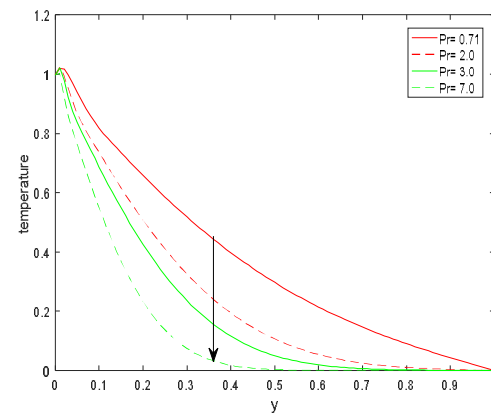


Figure4: Temperature profile for different fluid diffusivities ($t = 0.1, \delta = 2$)

Figures 2, 3 and 4 illustrate the variation in the fluid temperature for different values of δ , Pr and t respectively. Figure 2 illustrates that fluid temperature decreases as heat absorption increases while heat generation constitutes an increase in the fluid temperature within the channel, this trend is clearly obvious since heat generation acts to amplify the applied boundary heat so as to increase the temperature while heat absorption acts against the temperature to cause a decrease. It is obvious from Figure 3 that the temperature increases with time. The temperature gradient is also observed to be higher near the heated plate. As thermal diffusivity decreases, in Figure 4, the temperature is observed to decrease with it; this is because fluids with large Prandtl number have low thermal diffusivity which causes low heat penetration and reduced thermal boundary layer.

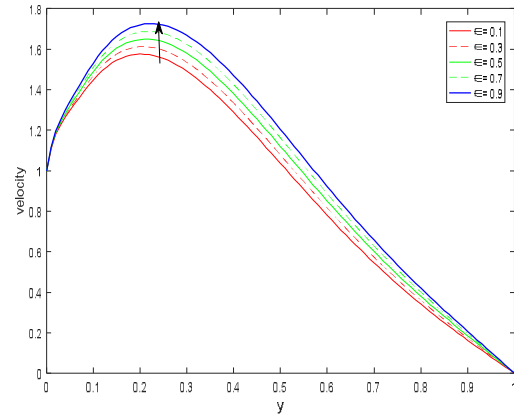


Figure 7: Velocity profile for different amplitude
 ($Gr=45, Pr=0.71, t=0.1, \omega = \frac{\pi}{18}, \delta = 3$)

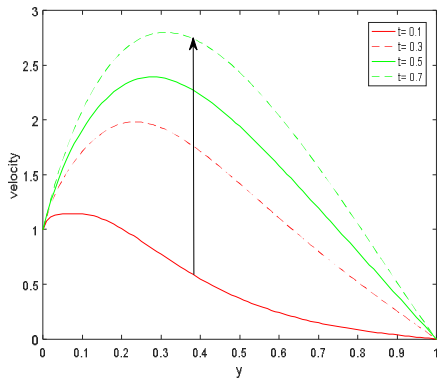


Figure 5: Velocity profile for different time
 ($Gr=45, Pr=0.71, \omega = \frac{\pi}{18}, \epsilon = 0.9, \delta = 3$)

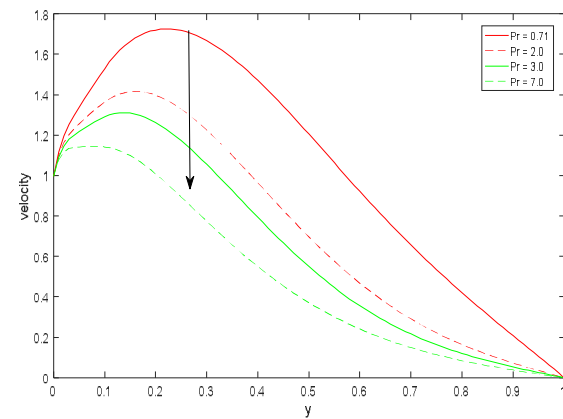


Figure 8: Velocity profile for different fluid diffusivities
 ($Gr=45, \epsilon = 0.9, t=0.1, \omega = \frac{\pi}{18}, \delta = 3$)

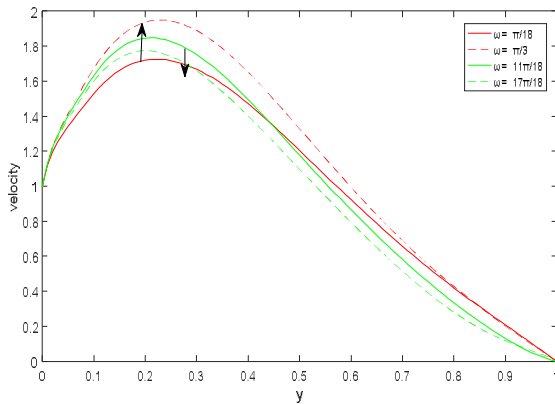


Figure 6: Velocity profile for different oscillating frequency
 ($Gr=45, Pr=0.71, t=0.1, \epsilon = 0.9, \delta = 3$)

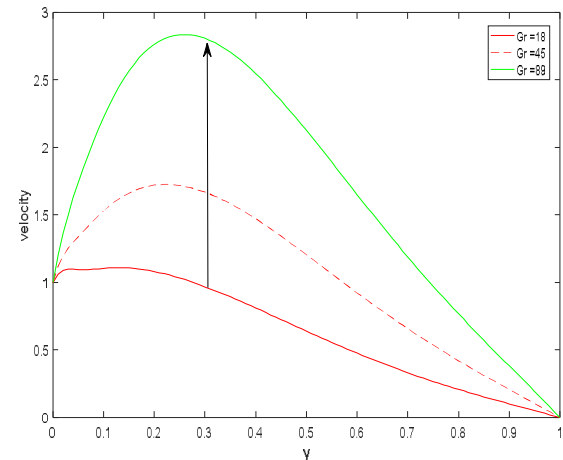


Figure 9: Velocity profile for different

convection current ($Pr=0.71, \epsilon=0.9, t=0.1, \omega = \frac{\pi}{18}, \delta = 3$)

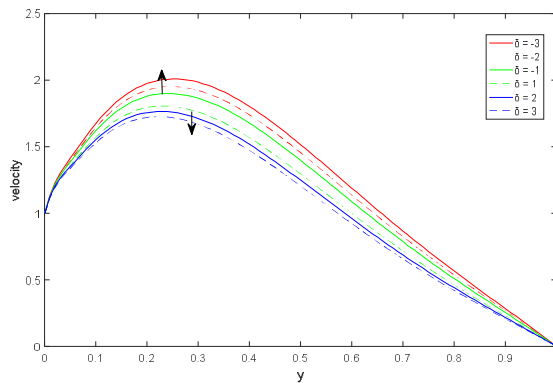


Figure10: Velocity profile for different heat generation and absorption ($Pr=0.71, \epsilon = 0.9, t=0.1, \omega = \frac{\pi}{18}, Gr=45$)

Figure 5 to 10 present the influence of the governing parameters on velocity profile. In Figure5, velocity is observed to increase with time. It is also asymmetrically distributed and skewed towards the heated plate because of the impulsive motion of one of the bounding plates. Figure6 presents that as the oscillating frequency increases, the velocity of the fluid increases and decreases at some point continuously (fluctuating) and also, at high frequency of the g-jitter, the effect of g-jitter on fluid velocity is high. Figure7 illustrates that as the amplitude increases, the velocity of the fluid increases and also as the amplitude is so small, the effect of g-jitter is so minimal, this is physically true since an increase in the amplitude of the fluctuating acceleration constitute an enhancement in the buoyancy of the fluid leading to an increase in the hydrodynamics within the channel. Figure8present that velocity decreases as the Prandtl number increases, this is hinged on the premise that an increase in the Prandtl number causes a decrease in the fluid temperature which in turn weakens the convection current and consequently decreases the momentum boundary layer. In Figure9, it was observed that as the Grashof number increases, the velocity of the fluid also increases, this is due to the physical fact that growing Grashof number increases the buoyancy of the working fluid which strengthens the convection current within the flow domain and hence an increase in fluid velocity is achieved. Figure10 reveals that velocity decreases as heat absorption increases, because as the heat absorption of the fluid increases, fluid temperature decreases causing a weakening in the convection current and hence the velocity, heat generation on the other hand is observed to cause an increase in fluid velocity.

Table1 presents the rate of heat transfer on the channel plates. The table shows that heat transfer increases on the heated and the cold plate with growing heat generation while it decreases on both plates with heat absorption. However, a decrease in the thermal diffusivity has been observed to increase the rate of heat transfer on the heated plate while it decreases it on the cold plate. This is attributed to the physical fact that a decrease in thermal diffusivity (increasing Pr) causes a decrease in fluid temperature which tends to increase the temperature difference between the heated plate and the fluid. The effect of time increase on rate of heat transfer is also depicted on the table. It can be deduced that the rate of heat transfer becomes lowered with growing time on the heated surface while it increases on the cold plate. This is due to the physical fact that fluid temperature increases with time which constitute a decrease or an increase in the temperature difference on the heated or cold plates respectively.

Table 1: rate of heat transfer for variation inPr, t and δ .

	δ	Pr=0.71		Pr=2.0	
		Nu ₁	Nu ₂	Nu ₁	Nu ₂
t=0.1	-3	5.08290	0.67571	7.772900	0.03864
	-1	4.61386	0.55947	7.510310	0.03551
	1	4.18810	0.46464	7.25641	0.03264
	3	3.79887	0.38706	7.01067	0.03001
t=0.3	-3	3.538271	0.59728	4.970250	0.73867
	-1	2.715111	0.13604	4.483430	0.60654
	1	2.07371	0.83260	4.04435	0.49983
	3	1.54997	0.62606	3.64514	0.41338

Table 2: The influences of the governing parameters regarding the skin-friction on the boundaries

	δ	Pr=0.71, $w = \frac{\pi}{18}$, Gr=45		Pr=2.0, $w = \frac{\pi}{18}$, Gr=45		Pr=0.71, $w = \frac{\pi}{3}$, Gr=45	
		τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
t=0.1 $\epsilon = 0.5$	-3	0.0258563	1.7097756	0.0258563	1.7097756	0.0258563	1.7097756
	-1	0.0258505	1.7097756	0.0259191	1.7097756	0.0259177	1.7097756
	1	0.0258429	1.7097756	0.0259176	1.7097756	0.0259161	1.7097756
	3	0.0258367	1.7097756	0.0259161	1.7097756	0.0259147	1.7097756
t=0.3 $\epsilon = 0.5$	-3	0.0281871	1.8997508	0.2925251	1.8997508	0.29201221	1.8997508
	-1	0.0280221	1.8997508	0.2914680	1.8997508	0.29098348	1.8997508
	1	0.0276259	1.8997508	0.2906568	1.8997508	0.29018860	1.8997508
	3	0.0270381	1.8997508	0.0290041	1.8997508	0.28958212	1.8997508
t=0.3 $\epsilon = 0.9$	-3	0.0281718	1.8997508	0.0292336	1.8997508	0.02914364	1.8997508
	-1	0.0280053	1.8997508	0.0291292	1.8997508	0.02904195	1.8997508
	1	0.0276087	1.8997508	0.0290481	1.8997508	0.02896336	1.8997508
	3	0.0270212	1.8997508	0.0289866	1.8997508	0.02890413	1.8997508

Table 2 above shows that skin-friction on the heated plate increases with growing heat generation and remains stable on the cold plate, while heat absorption brought about decrease of the skin-friction on the heated plate and also the cold plate remains stable, the skin-friction on the cold plate is higher than that of the heated plate. As thermal diffusivity

decreases (i.e. increasing Pr) it was observed that the skin-friction on the heated plate increases while that of the cold plate remains stable.

The effect of time, oscillating frequency of g-jitter and g-jitter amplitude is also depicted on the table and it was observed that as the time increases the skin-friction on the heated and cold plate also

increases, and increasing the amplitude of the g-jitter brought about decrease in the skin-friction of the heated plate while that of the cold plate remains stable. Increase in the g-jitter oscillating frequency brings about decrease in the skin-friction on the

heated plate and that of the cold plate remains stable. The stability in the figure above is due to inviscid region i.e. the region where the frictional effects is negligible and the velocity remains essentially constant.

Table 3: The influence of convection current in the flow domain skin-friction on the boundaries ($t=0.1, \omega = \frac{\pi}{18}, Pr=0.71, \delta = 2, \epsilon = 0.9$)

Gr	τ_0	τ_1
18	0.0258890	1.7097757
45	0.0258381	1.7097757
89	0.0257550	1.7097757

Table3 above presents the effect of the convection current on the skin-friction in the flow domain, and it was observed from the table that the skin-friction on the heated plate is lower

than that of the cold plate, and also as the convection current increases the skin-friction on the heated plate decreases while that of the cold plate remains stabled,

Table 4: The mass flow rate and the average temperature in the flow domain for variation in Pr, t, δ, ϵ and ω

		Pr=0.71, $w = \frac{\pi}{18}$, Gr=45		Pr=0.71, $w = \frac{\pi}{18}$, Gr=45		Pr=0.71, $w = \frac{\pi}{18}$, Gr=45	
	δ	Q	θm	Q	θm	Q	θm
t=0.1	-3	1.1635829	0.193210	0.7234420	0.0179330	0.76746050	0.0174605
	-1	1.0952767	0.0195431	0.7107939	0.0179524	0.7541581	0.0174653
	1	1.0355988	0.0197338	0.6987780	0.0179675	0.7414679	0.0174674
	3	0.9832472	0.0198965	0.6873566	0.0179784	0.7293562	0.0174667
$\epsilon = 0.5$	-3	3.5847510	0.0696127	2.2157400	0.0495340	1.8662234	0.0582764
	-1	2.9128586	0.0703036	2.0445402	0.0500763	1.7531792	0.0581517
	1	2.4529176	0.0703325	1.8971512	0.0505419	1.6545869	0.0579635
	3	2.1263481	0.0698965	1.7697075	0.0509318	1.5682517	0.0577183
t=0.3	-3	4.3649790	0.0630221	2.5593851	0.0458672	1.9302533	0.0598864
	-1	3.4495078	0.0646107	2.3470731	0.0470432	1.8226233	0.0594356
	1	2.8359371	0.0654737	2.1645519	0.0470432	1.7279361	0.0589508
	3	2.4091807	0.0657631	2.0069769	0.0475348	1.6443564	0.0584361
$\epsilon = 0.9$	-3	4.3649790	0.0630221	2.5593851	0.0458672	1.9302533	0.0598864
	-1	3.4495078	0.0646107	2.3470731	0.0470432	1.8226233	0.0594356
	1	2.8359371	0.0654737	2.1645519	0.0470432	1.7279361	0.0589508
	3	2.4091807	0.0657631	2.0069769	0.0475348	1.6443564	0.0584361

Table4 above shows the effect of the governing parameters on the mass flux and Mean temperature within the channel. It was observed from the table that as the heat generation is growing, the mass flux

increases and Mean temperature decreases while the mass flux decreases and Mean temperature increases with growing heat absorption, this is due to the fact that an increase in heat absorption leads to a decrease

in temperature, a corresponding decrease in velocity and hence, the mass flux. Mass flux and Mean temperature increases with growing time, this is true because increase in time increases the temperature and the velocity of the fluid in the flow domain and hence increases the mass flux and the Mean temperature. Also as thermal diffusivity decreases the mass flux and Mean temperature decreases, this is due to the fact that decrease in thermal diffusivity brings about decrease in the temperature and velocity of the fluid and hence decreases the Mean temperature and mass flux. As the oscillating frequency of the g-jitter increases, the mass flux

increases while the Mean temperature decreases. And also as the amplitude of the g-jitter increases it was observed that the mass flux increases and Mean temperature decreases. It is interesting to note that the influence of frequency of oscillations of g-jitter on mass flux and Mean temperature depends on time. For instance for a relatively small time, an increase in the frequency of oscillation causes an increase in mass flux with a decrease in the Mean temperature. However, for a relatively large time the trend is reversed in which a decrease in mass flux and an increase in Mean temperature are the resultant effects of increase in the frequency of oscillation of g-jitter.

Table 5: Variation in convection current on mass flux and Mean temperature

Gr	$t=0.1, \omega = \frac{\pi}{18}, Pr = 0.71, \delta = 2, \epsilon = 0.9$	
	Mass flux	Mean temperature
18	0.6384696	0.0245840
45	1.0694436	0.0190273
89	1.7717716	0.0157641

Table 5 above shows the influence of the convection current on the mass flux and Mean temperature and it was observed that as the convection current is growing the mass flux grows while the Mean temperature decreases, this is because the impulsive motion of one of the bounding plates acts in support of upward convection currents thereby boosting the mass flux and decreasing the Mean temperature.

Table 6: Comparison of the present work with Jha and Ajibade[32]

δ	Jha and Ajibad (2010) Pr = 0.71, t = 0.3, Gr = 45, MASS FLUX	Present work without the effect of g-jitter Pr = 0.71, t = 0.3, Gr = 45, w = 0, E = 0 MASS FLUX	Present work without the effect of g-jitter Pr = 0.71, t = 0.3, Gr = 45, $W = \frac{\pi}{18}$ MASS FLUX	Percentage increase %increase = ((new number - original number)/original number) X 100
-4	2.8461494528	2.8461494528	4.9567340314	74.1%
-3	2.6094661812	2.6094661753	4.3649790621	67.27%
-2	2.4105796213	2.4105796596	3.9458063341	63.69%
-1	2.2420472412	2.2420472719	3.4495078817	53.86%
1	1.9741432178	1.9741432327	2.8359371317	43.65%
2	1.8666645719	1.8666645125	2.5489348418	36.55%

3	1.7728072432	1.7728072552	2.4091807361	35.90%
4	1.6903053714	1.6903053875	2.2347671442	32.21%

Table 6 shows the numerical comparison between the work of Jha and Ajibade (2010) and the present work by running a numerical value on the effect of heat generation/absorption parameter on the mass flux. From the table above we can discover that the numerical values in the work of Jha and Ajibade[32] agrees up to the seventh decimal places with the present work when the amplitude and the oscillating frequency of the present work were being relaxed by setting them to zero, which shows a perfect agreement with the work of Jha and Ajibade[32] and also comparing the work of Jha and Ajibade (2010) with the present work which have the effect of g-jitter, it was found out that there is a percentage increase.

4 CONCLUSION

5 NOMENCLATURE

c_p - Specific heat of the fluid at constant pressure

g - Acceleration due to gravity [ms^{-2}]

H - Gap between the plates

Pr - Prandtl number

u' - Dimensional fluid velocity [ms^{-1}]

u - Dimensionless fluid velocity

\vec{u} - Dimensionless fluid velocity in Laplace domain

U - Velocity of the plate at $y' = h$ for $t' > 0$ [ms^{-1}]

This paper has considered a g-jitter effect on transient natural convection Couette flow in a vertical channel. The motion is induced by the impulsive motion of one of the channel plates along with by the asymmetric heating of the plates. The influence of the governing parameters on the temperature, velocity, rate of heat transfer, Nusselt number, skin-friction, mass flux and Mean temperature are discussed with the aid of graphs and numerical values. The study concluded that heat generation increase causes an increase in the mass flux and Mean temperature while heat absorption acts in the reverse. Mass flux and mean temperature increases with time. As the amplitude of the g-jitter increases the mass flux increases and the mean temperature decreases. Also the influence of g-jitter fluctuation frequency on flow formation and thermodynamic is strongly dependent on time.

t - Dimensionless time

t' - Dimensional time [s]

Gr - Grashof number

T_0 - Initial temperature of the fluid and plates at $y' = h$ [k]

T_w - The temperature of the plate at $y' = 0$ [k]

T' - Dimensional temperature of the fluid [k]

y' - Dimensional coordinate perpendicular to the plate [m]

x' - Dimensional coordinate parallel to the plate [m]

y - Dimensionless coordinate perpendicular to the plate

g^* - Fluctuating acceleration due to gravity (g-jitter) [ms^{-2}]

h - Width of the channel [m]

Greek alphabets

β - Coefficient of thermal expansion [k^{-1}]

κ - Thermal conductivity

μ - Coefficient of viscosity

ν - Kinematic viscosity [m^2s^{-1}]

δ - The heat generating/absorbing

ω - oscillating frequency

ϵ - Amplitude

θ - Dimensionless temperature of fluid

$\vec{\theta}$ - Dimensionless temperature of fluid in Laplace domain

6 APPENDIX

The constant used to define temperature, velocity, Nusselt number, skin-friction and Mean temperature

$$C_1 = \frac{e^{-2m}}{s(1 - e^{-2m})}$$

$$C_2 = \frac{1}{s(1 - e^{-2m})}$$

$$\bar{u}_{p0} = a_0 + a_1 + a_2 + a_3 + a_4 + a_5$$

$$\bar{u}_{p01} = a_0 e^m + a_1 e^{-m} + a_2 e^{m_1} + a_3 e^{-m_1} + a_4 e^{m_2} + a_5 e^{-m_2}$$

$$C_4 = \frac{1}{s(1 - e^{-2m^p})} + \frac{\bar{u}_{p01} e^{-m^p}}{(1 - e^{-2m^p})} - \frac{\bar{u}_{p0}}{(1 - e^{-2m^p})}$$

$$C_3 = -C_4 e^{-2m^p} - \bar{u}_{p01} e^{-m^p}$$

$$m = \sqrt{\frac{s + B}{A}}$$

$$m^p = \sqrt{s}$$

$$m_1 = \sqrt{\frac{p + B}{A}}$$

$$m_2 = \sqrt{\frac{q + B}{A}}$$

$$p = s - i\omega$$

$$q = s + i\omega$$

$$A = \frac{1}{Pr}$$

$$B = \frac{\delta}{Pr}$$

$$a_0 = \frac{-GrC_1}{m^2 - s}$$

$$a_1 = \frac{-GrC_2}{m^2 - s}$$

$$a_2 = \frac{-GrC_1 \epsilon}{2i(m_1^2 - s)}$$

$$a_3 = \frac{-GrC_2 \epsilon}{2i(m_1^2 - s)}$$

$$a_4 = \frac{GrC_1 \epsilon}{2i(m_2^2 - s)}$$

$$a_5 = \frac{GrC_2 \epsilon}{2i(m_2^2 - s)}$$

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