Minimizing PAPR of OFDM Signal Using Discrete Cosine Transform and Log Companding

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ABSTRACT

This paper proposed precoding of OFDM signal using Discrete Cosine Transform (DCT) and adding of Log companding algorithm so as to reduce PAPR. Three scenarios were considered to fully examine the effect of PAPR in OFDM system based on simulations conducted in MATLAB environment while varying the number of subcarriers, N = 64 and 256. Initial simulation using conventional OFDM showed high PAPR in transmitted signal for N = 64 and 256, which resulted in PAPR of 9.741 and 11.41 dB respectively. In the second case, log companding technique was introduced into the OFDM and it was observed that PAPR was significantly reduced to 5.664 dB, and 6.101 dB by a percentage improvement of 41.9% and 46.5% for N = 64 and 256. For the proposed scheme, DCT scheme was used to initially precode the OFDM and a log companding algorithm applied, the results obtained indicated more than half level of reduction in PAPR of OFDM signal such that for N = 64 and 256 the PAPR was reduced to 4.214 dB and 4.743 dB, which corresponded to percentage improvement of 56.7% and 58.4% respectively.

Keywords - **Discrete Cosine Transform, Log companding, OFDM, PAPR**

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I. INTRODUCTION

With increasing development in digital wireless communication in recent times, the need for high-speed data transmission has grown rapidly. Orthogonal frequency division multiplexing (OFDM) methods are presently being implemented to sustain the demand for more communication capacity. Also, implementing OFDM different antenna configurations at the transmit and receive ends so as to increase the diversity gain and/or to improve system capacity on time variant and capacity and frequency selective channels, which lead to MIMO structure.

Several techniques such as precoding, repeated clipping and Filtering (RCF), coding, tone injection, peak windowing, selected mapping (SLM), and partial transmit sequence (PTS), and companding have been proposed to mitigate the high PAPR associated with OFDM signals. However, there are still concerns on computational complexity due to increase number of subcarriers (or sub-blocks) including increase in number of iterations or evaluation processes.

This paper proposes a discrete cosine transform enhanced Log compading (DCT-Log) algorithm to reduce PAPR in OFDM signal. The proposed technique does not require complex computation and as such does not lead to increase in system complexity. It ensures that the phases of the OFDM signal are maintained while compressing large signals. Log companding technique is considered the best practical companding algorithm (Mounir and El_Mashade, 2019). It offers better PAPR reduction performance. There is no need for side information at the receive end and as such offers better bit error ratio (BER) performance. Also, it offers better out-of-bound (OOB) radiation (Mounir and El_Mashade, 2019).

II. EMPIRICAL REVIEW

In this section, some of the current studies on PAPR reduction in OFDM not more than ten years of publication from this present study are empirically reviewed.

Agwah et al. (2020a) attempted to address the problem of PAPR in OFDM system using Discrete Fourier Transform (DFT) precoding plus Repeated Clipping and Filtering (RCF). The study combined DFT and RCF algorithms to minimize PAPR. The algorithms were introduced into conventional OFDM system. Simulation analysis conducted in MATLAB revealed that the proposed scheme yielded 46.91% improvement in PAPR reduction of the OFDM signal after four iteration process. The problem with this approach using RCF is that it is time consuming as well as requiring complex computation.

Agwah et al. (2020b) carried out optimization of PAPR in MIMO-OFDM Long Term Evaluation (LTE) network. In the study, a hybrid technique using DFT precoded conventional OFDM signal that combines RCF and Mu-law companding algorithm was used to reduce PAPR in OFDM system. The block diagram of the developed hybrid scheme for optimizing PAPR in OFDM signal is shown in Figure 2.5. Simulations were conducted for LTE network considering 256 data points with QPSK modulation technique. The results obtained revealed that the conventional OFDM signal has PAPR of 10.51 dB and was reduced 1.757 dB, which is an improvement by 83.28% in PAPR value.

Ekengwu et al. (2020) improved the PAPR in OFDM signal using Discrete Cosine Transform (DCT) precoding with RCF plus Mu-law companding techniques. The study carried out four analysis steps before eventually implementing the proposed scheme, which include: conventional or uncoded OFDM, introduction of RCF into conventional OFDM system, precoded OFDM with RCF, and precoded OFDM with RCF plus Mu-law. The simulation results revealed that proposed technique reduced PAPR in OFDM signal by 80.36%.

Amhaimar et al (2018) proposed firework algorithm (FWA) enhanced partial transmit sequence (PTS) technique. The FWA is a swarm intelligent algorithm implemented to reduce PTS computational complexity. The results obtained from the simulation conducted in MATLAB revealed the adequacy and the effectiveness of the proposed method which can effectively reduce the computation complexity while keeping good peak-to-average power ratio (PAPR) reduction. Moreover, it turns out from the results that the proposed PTS scheme-based FWA clearly outperforms the hottest and most important evolutionary algorithm in the literature like simulated annealing (SA), particle swarm optimization (PSO), and genetic algorithm (GA). However, the proposed FWA was tested on IEEE 802.11a and 802.16e standards.

Amhaimar et al. (2019) used New Swarm Intelligence Algorithm in OFDM systems to reduce PAPR. The objective was to optimization peak to average power ratio (PAPR) in OFDM system. The study formulated the phase factors searching of PTS technique as a particular combination optimization problem processed by the Fireworks Algorithm, to search the optimal combination of phase factors for PTS without degradation in PAPR performance and with minimum complexity. The simulation results among the FWA-PTS, genetic algorithm (GA)-PTS, simulated annealing (SA)-PTS, standard particle swarm optimization (SPSO)-PTS, selective mapping (SLM) and conventional PTS showed that FWA-PTS has a promising performance in terms of both optimization accuracy of PAPR and convergence speed over conventional optimization algorithms for IEEE 802.11a (Wireless LAN) standard.

Anoh et al. (2017) reduced PAPR in precoded OFDM signals using root-based nonlinear companding. The study takes advantage of the fact that the amplitudes of PAPR signals are nonlinearly and non-monotonically increasing to apply roots to the amplitude distribution so as to change the probability density function (PDF) and therefore reduces the PAPR. This approach was demonstrated by applying it on standard Mu-law companding technique in reducing PAPR of OFDM signals. The choice of the Mu-law companding technique was because it known to expand the amplitudes of low power signals only without affecting the signals of higher amplitude. The authors called the technique root-based Mu-law companding (RMC). The scheme was able to simultaneously expand and compress OFDM signal amplitudes unlike the Mu-law. Furthermore, a second transformation that did not depend on the Mu-law comapnding (MC) model was proposed. The results of the two proposed scheme was found to outperform four other commonly used companding scheme like MC, log-based modified (LMC), hyperbolic arc-sine companding (HASC) and exponential companding (EC). In addition, the OFDM signal was precoded using Discrete Hartley Transformation (DHT) in order to further reduce the PAPR limits achieved by RMC by distorting phase. Maintaining the BER, the DHT-precoded RMC was observed to perform better than all four other companding schemes such as MC, EC, HASC, and LMC in terms of PAPR reduction of OFDM signal.

Manjula and Muralidhara (2017) combined repeated clipping and filtering (RCF) and selective mapping (SLM) techniques in OFDM system to reduce PAPR. Clipping and filtering applied to OFDM system and simulated using MATLAB. Simulation analysis show that for single clip and filtering with variation of clipping ratios (CR) equal to 0.4,1,2 and 4, decreasing CR caused BER to decrease. The use of RCF and SLM was found to reduce the PAPR to considerable level.

Dubey and Gupta (2016) studied PAPR reduction precoding techniques with and without repeated clipping and filtering method (RCF). The study showed that precoding techniques together with repeated clipping and filtering (RCF) technique provided important improvement in PAPR reduction than using only precoding techniques. Results from the simulation conducted revealed that clipping gives better performance than using precoding with and without repeated clipping and filtering with oversampling factor of 2.

Mounir et al. (2018) examined different precoding matrices for PAPR reduction in OFDM systems. Six precoding matrices namely Walsh-Hadamard Transform (WHT), Discrete Cosine Transform (DCT), Zadoff-Chu Matrix Transform (ZCT), Discrete Hartley Transform (DHT), Discrete Fourier Transform (DFT), and Square-root Raised Cosine (SRC), found in the literature were compared. Simulation results indicated that, all precoding matrices, except the one which is based on square-root raised cosine function (SRC), were not effective in terms of BER performance in presence of nonlinear power amplifier (PA), especially in high modulation order schemes such as 16-QAM and 64-QAM. On the other hand, precoding technique based on SRC matrix needed high data rate loss to improve the BER performance in presence of nonlinear PA. Thus, the authors stated that precoding technique aggravated the problem especially in high modulation order techniques, except the SRC matrix-based precoding technique, which cannot be used without high data rate loss. ZCT and DFT precoding matrices were found to have the best PAPR reduction gain among the six (N × N) predefined matrices.

Mounir and El_Mashade (2019) examined the performance of six companding transforms: mu-law, Exponential, Tanh, Log, linear symmetric transform (LST), and linear asymmetric transform (LAST). The performance of the companding algorithms were compared in terms of complementary cumulative distributive function (CCDF), bit error ratio (BER), and power spectral density (PSD). The modulation technique used in the study was 16-QAM over AGWN channel with 256 subcarriers and 192 data carriers. The oversampling factor used was 4. The simulation results showed that Tanh and Log companding techniques kept small signals unchanged, while compressing large signals and are better than others in terms of BER performance. The Log companding yielded better PAPR than Tanh. The study concluded that Log companding transform is the best practical compounding transform among the others.

An empirical review of most recent studies on PAPR reduction has also been presented. In this paper, precoding algorithm of discrete cosine transform (DCT) was added with Log companding technique in OFDM system. Also quadrature phase-shift keying (QPSK) modulation over Rayleigh channel is considered in this work. The choice of Rayleigh channel over AGWN channel in this work is due to the fact that AGWN channel is an ideal (or standard) channel used for analysis purpose only whereas in Rayleigh fading channel the effect of multipath such as constructive and destructive interference, and phase shift of the signal is taken into considering.

III. SYSTEM DESIGN

This chapter presents the methods for achieving the objectives of the work. It is divided into peak to average power ratio (PAPR) problem in OFDM system, proposed OFDM model, and simulation parameter.

3.1 Peak to Average Power Problem in OFDM System

At each antenna of OFDM system, the peak-to-average power ratio (PAPR) is given by (Amhaimar et al, 2018):

$$\mathrm{PAPR}\{x_i\} = \frac{\max\{|x_i(n)|^2\}}{E\{|x_i(n)|^2\}}, \quad 0 \le n \le LN-1, \tag{1}$$

where i = 1, 2… N_T number of transmit antennas and L is the oversampling factor (OF). The time domain signal at each transmit antennas can be presented by:

$$x_i(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k^i e^{(j2\pi nk/LN)} \tag{2}$$

The expression that characterizes the peak power variation of MIMO-OFDM systems is defined as:

$$\mathrm{PAPR}_{\mathrm{MIMO-OFDM}} = \max\{\mathrm{PAPR}(x_i)\}, \; i = 1,....., N_T \tag{3}$$

Usually, PAPR of OFDM signal is analyzed using statistical tool (probability distribution function). This probability distribution function in the analysis of peak power of signal in modern communication is Complementary Cumulative Distribution Function (CCDF) measurement Manjula and Muralidhara, (2017). In practice, CCDF is taken as the measurement index for the probability of PAPR greater than a threshold (Yi and Iinfeng, 2009) and it is given by Ekengwu et al. (2020):

$$\begin{aligned} P(\mathrm{PAPR} > z) &= 1 - P(\mathrm{PAPR} \le Z \\ &= 1 - f(Z)^N = 1 - (1 - \mathrm{EXP}(-Z))^N \end{aligned} \tag{4}$$

where N is statistically independent uncorrelated signal samples.

3.2 Proposed OFDM System

The structure of OFDM system proposed in this work is shown in Fig. 1. It is an OFDM system in which the OFDM signal is coded with Discrete Cosine Transform (DCT) precoding algorithm and integrated with Log companding technique. These algorithms are discussed in this section.

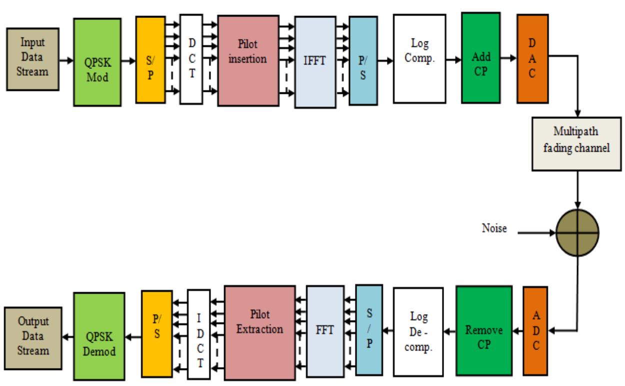

Fig. 1 Proposed OFDM model

3.2.1 DCT Precoding

The precoding technique involves multiplying the modulated data of each OFDM block in frequency domain by a precoding matrix **P** before the IFFT process. The reverse process takes place at receiver where the inverse of the precoding matrix $\mathbf{P}^{-1}$ is used after FFT operation. However, in this paper, the focus is on the transmitter where PAPR performance needs to be optimized for improved transmitted signal. The baseband modulated data stream is grouped into blocks of length $(N-N_p)$ symbols each (Mounir et al., 2018). An already established $N\times(N-N_p)$ precoding matrix $\mathbf{P}$ is used to multiply each block of symbols. The $\mathbf{P}$ matrix is given by:

$$P=\begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,(N-N_p)} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,(N-N_p)} \\ \vdots & \vdots & \cdots & \vdots \\ P_{N,1} & P_{N,2} & \cdots & P_{N,(N-N_p)} \end{bmatrix} \quad (5)$$

where $P_{n,m}$ are the elements of the precoding matrix, N is the number of subcarriers, and $(N-N_p)$ denotes the data block length prior to precoding with $0\le N_p<N$. The precoding matrix becomes $(N\times N)$ matrix when $N_P=0$ and the rate loss reduces to zero (Mounir et al., 2018)

The expression for the DCT precoding technique, which is a $(N\times N)$, is given by:

$$P_{n,m}=\begin{cases} \sqrt{\frac{1}{N}}, & n=0 \quad \text{and} \quad 0\le m\le(N-1) \\ \sqrt{\frac{2}{N}}\cos\left(\frac{2\pi\pi n+\pi n}{N}\right), & 1\le n\le(N-1) \quad \text{and} \quad 0\le m\le(N-1) \end{cases} \quad (6)$$

3.2.2 Logarithm Function Companding

In companding algorithm, OFDM signal is compressed at the transmitter while it is expanded at the receiver. Logarithm function companding or simply Log companding is used in this work. The Log companding algorithm is defined by Mounir and El_Mashade (2019):

$$C(x)=\begin{cases} x_n, & |x_n|\le x_{th} \\ K_1\mathrm{Log}_e\left[1+(x_nK_2)\right], & |x_n|>x_{th} \end{cases} \quad (7)$$

where x_{th} is the threshold, before compression is performed. K_1 and K_2 are positive integers controlling the level of compression or amount of companding such that $0\le K_1K_2\le1$.

3.3 Simulation Parameter

The effectiveness of the proposed PAPR reduction algorithm in OFDM system is analyzed using the following test parameters used in the simulation test conducted by Agwah et al. (2020a and 2020b) for LTE network given in Table 1.

Table 1 LTE Simulation Test Parameter (Agwah et al., 2020a and 2020b)

Parameter	Description /value
Modulation	QPSK
FFT Size	256
Spacing	15KHz
Band Width (BW)	1250KHz
Cyclic Prefix (CP)	1/4 of FFT Size
Number of Symbol (nsym)	1×10^3
Sampling Frequency (f_s)	192MHz
Sampling Period (T_s)	192µs
Max. Doppler Frequency Shift (F_{Dmax})	0.01Hz

IV. RESULTS AND ANALYSIS

Three basic scenarios were considered during the simulation according to the objectives of the work namely: simulation of conventional orthogonal frequency division multiplexing (OFDM), simulation of OFDM with logarithm companding algorithm, and simulation of precoded OFDM with logarithm companding algorithm (proposed system) that involve the variations of different number of subcarriers (N) to validate the proposed peak to average power ratio (PAPR) reduction scheme For the analysis, performance evaluation is carried out using the complementary cumulative distributive function (CCDF). According to the simulation parameters listed in Table 1, optimum parameter of k values (k_1 = 1, k_2 = 1) stated in Mounir and El_Mashade (2019) for the logarithm companding technique were used in this work. The number of subcarrier, N = 64 and 256 were used to validate the effectiveness of the proposed scheme under varying N conditions. Also, it should be noted that the analysis of the simulation has been limited to the transmit section of the OFDM model which is the part of the system where PAPR effect arises during wireless communication involving OFDM system.

The graphs of the conventional OFDM system for N= 64 and 256 are shown in Fig. 2 and 3. In this scenario, the OFDM system was simulated assuming no PAPR reduction scheme was included as part of the transmit process. This was considered as a reference that helps to determine the actual value of the OFDM system at its conventional state for different values of N.

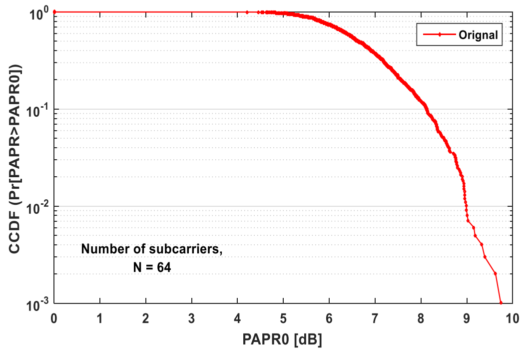

Fig. 2 PAPR of conventional OFDM for N = 64

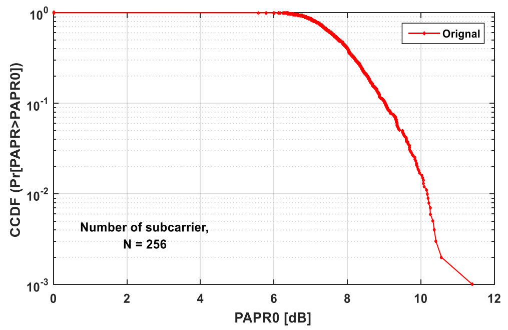

Fig. 3 PAPR of conventional OFDM for N = 256

The numerical analysis of simulation plots for various number of subcarriers shown in Fig. 2 and 3, show that for N = 64, the PAPR was 9.741 dB and for N = 256, the PAPR was 11.41dB at CCDF 10^{-3}.

Now for N = 64 and 256 adding logarithm companding scheme as part of the OFDM system to examine, the resulting effect of PAPR in the modulated signal that is to be transmitted as shown in Fig. 4 and 5.

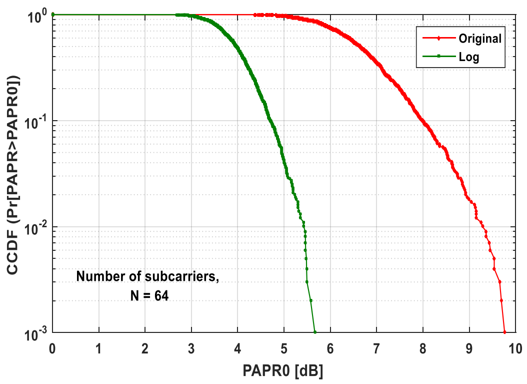

Fig. 4 PAPR of conventional OFDM with log companding for N = 64

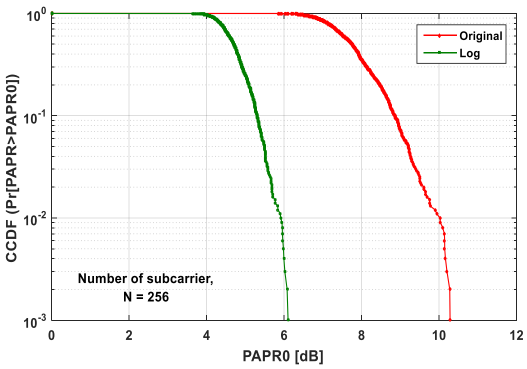

Fig. 5 PAPR of conventional OFDM with log companding for N = 256

In order to analyze the performance improvement of the OFDM system, the expression in (8) used in Agwah et al. (2020a and 2020b) is employed and is given by:

$$\%PAPR_{improvement} = \frac{PAPR_{OFDM\_Without} - PAPR_{OFDM\_With}}{PAPR_{OFDM\_Without}} \times 100$$

(8)

where $PAPR_{OFDM\_Without}$ is the PAPR value for conventional OFDM system, $PAPR_{OFDM\_With}$ is the PAPR value with log companding or with precoding plus log algorithm. The numerical analysis of simulation plots for various number of subcarriers shown in Fig. 4 and 5 , revealed that the least value (5.664 dB) reduction of PAPR in OFDM signal is obtained when N = 16, while the value (6.101 dB) of reduction is achieved when N = 256. However, in terms of percentage improvement with respect to the original value of conventional OFDM signal, 41.9% was provided at N = 64 while 46.5% was yielded by N = 256. Generally, it can be seen that the addition of log companding algorithm significantly reduced PAPR in OFDM system.

The simulation results in terms of the proposed system using discrete cosine transformation (DCT) with log companding (DCT + Log) are presented in this subsection considering various number of subcarriers as shown in Fig. 6 and 7.

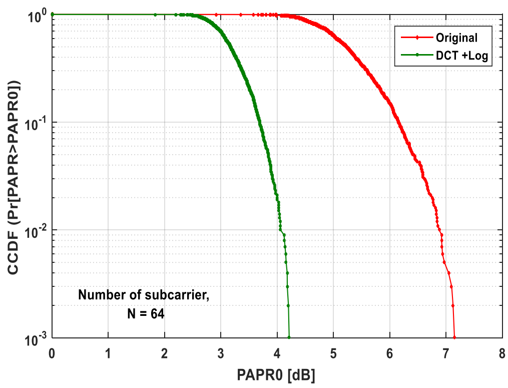

Fig. 6 PAPR DCT precoded OFDM with log companding for N = 64

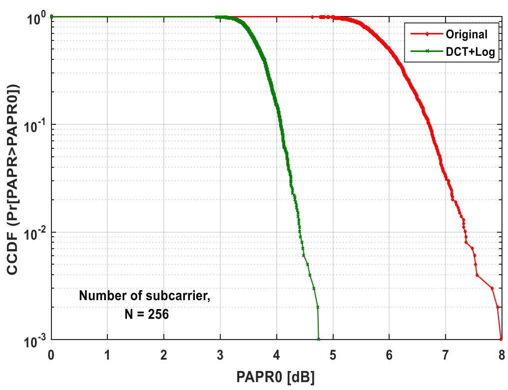

Fig. 7 PAPR DCT precoded OFDM with log companding for N = 256

The numerical values of the resulting PAPR from the precoded OFDM enhanced log companding technique proposed in this work based on the analysis of simulation plots of various number of subcarriers shown in Fig. 6 and 7, shows that a PAPR value of 4.214 dB reduction of PAPR in OFDM signal is obtained when N = 64, while value of 4.743 dB reduction was achieved when N = 256. However, in terms of percentage improvement with respect to the original value of conventional OFDM signal, 56.7% was provided at N = 64 while N= 256 yielded 58.4%. Generally, it can be seen that the by using DCT precoding scheme with log companding algorithm, a significant reduction of PAPR in OFDM system can be achieved with more than half percentage improvement.

The summary of the results obtained from the MATLAB simulation performed to examine and evaluate the performance of DCT precoded OFDM signal integrating a log companding algorithm to reduce PAPR level in OFDM revealed the following findings which are conspicuously evident from the simulation plots (Fig. 2 through 7): (a) Increase in the number of subcarriers can cause increase in PAPR value in OFDM system. (b) The use of log companding technique provides significant reduction of PAPR value of OFDM signal. (c) Also, by initially coding the OFDM signal with precoding technique such as DCT used in this work and then introducing log companding technique can provide even more significant level of reduction of PAPR in OFDM system.

V. Conclusion

In this paper, the proposed DCT precoded OFDM system uses log companding technique to reduce the PAPR of OFDM signal. In this approach, the data stream to be transmitted is fed into transmitted of an OFDM system and it is initially precoded using DCT precoding matrix and then the log companding acts on it by compressing the coded OFDM signal and thereby eliminating the peaking effect of the signal that can push the transmitter amplifier into nonlinearity that can caused signal distortion. The mathematical equations representing the model of the various techniques considered for examining PAPR in OFDM system and subsequent reduction of PAPR of OFDM signal were have been presented. The study presented three models of OFDM systems namely, conventional OFDM, log companding enhanced OFDM, and DCT plus log companding enhanced OFDM for PAPR reduction. The simulation results showed that for the various number of subcarriers considered, the proposed algorithm was able to provide significant reduction of PAPR in OFDM system. Generally, the proposed scheme (DCT plus log companding) has shown better efficient performance in PAPR reduction than existing technique (conventional OFDM plus log companding).

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