Minimizing Peak to Average Power Ratio in OFDM System with WHT and Log Companding

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------ABSTRACT-----

Orthogonal Frequency Division Multiplexing (OFDM) suffers the effect of Peak to Average Power Ratio (PAPR) that reduces the strength or power of radio frequency (RF) signal at the transmitter and which also pushes the performance of transmitter amplifier into distortion. In order to minimize the effect of PAPR in OFDM, this paper proposed a system that precodes the OFDM signal using Walsh-Hadamard Transform (WHT) and Log function companding algorithm. Simulations were conducted in MATLAB environment while varying the number of subcarriers, N = 16, 64, 128, 256. simulation results indicated original OFDM revealed high PAPR in transmitted signal for the different number of subcarriers (N) such that for N =16, 64, 128, and 256, resulting in 7.964dB, 8.885 dB, 9.548 dB, and 9.961 dB respectively. With the introduction of WHT-Log companding scheme, the PAPR of OFDM signal was significantly reduced to 4.775 dB, 4.285 dB, 4.517 dB, and 4.602 dB by a percentage improvement of 40%, 51.8%, 52.7%, and 57.8% for N = 16, 64, 128, and 256 respectively. As a result, from the simulation results, the proposed system can be said to have largely reduced PAPR in OFDM system using a scheme with significant computational complexity.

Keywords - Log companding, OFDM system, PAPR, WHT

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I. INTRODUCTION

Advances in wireless communication technology have resulted in increasing demand for standards with improved and promising capabilities. This has brought about the need for available wireless spectrum capacity to be improved. With the purpose of meeting the demands of customers, many technologies have been developed. Examples of such technologies are Wi-Fi (IEEE 802.11n) networks, Long Term Evaluation (LTE) networks (4G, 3GPPLTE) and other radio and wireless communication networks (Such as WiMAX and HSPA⁺) to offer increased link capacity, spectral efficiency, and data rate [1].

One technology that has gain popularity in providing enhanced wireless communication is Orthogonal Frequency Division Multiplexing (OFDM). The addition of OFDM corrects the multipath effect by efficiently using the spectrum in overlapping the sub-carriers. This therefore provides increase data rate, reduce Inter Symbol Interference (ISI) and then utilizes the spectrum efficiently that is needed for video transmission and other multimedia messages.

The OFDM system output consists of many subcarriers that are superimposed, which results in condition where some instantaneous power outputs could largely increase and become a lot higher than the average power of the system when the carriers are in the same phase. This is called peak to average power ratio (PAPR) and it is a key problem in OFDM system. Power amplifiers with high power capacity are required to transmit OFDM signals with high PAPR. Nevertheless, power amplifiers of these kinds are very costly and have low cost effectiveness. If the peak power of the OFDM signal is too high, it can shoot above the amplification capacity of the linear power amplifier and thereby giving rise to nonlinear distortion that alters the superposition of the signal bandwidth and subsequently caused performance degradation. If no actions are put in place to reduce PAPR of OFDM signal, serious restriction could be place on its practical implementation.

This paper proposes a Walsh-Hadamard Transform coded OFDM with Log compading (WHT-Log) algorithm to minimize PAPR in OFDM signal. There computation is no computational complexity associated with the proposed scheme. Hence, it will not cause increase in system complexity. It guarantees that the phases of the OFDM signal are preserved while compressing large signals. Log companding. According to [2], offers the best practical companding algorithm. It gives better PAPR performance. With log companding, no need for side information at the receive end therefore providing better bit error rate (BER) performance. Also, it offers better out-of-bound (OOB) radiation [2].

II. OFDM SYSTEM

Orthogonal Frequency Division Multiplexing (OFDM) system employs a multicarrier modulation scheme for transmitting a single data stream over a number of lower rate orthogonal subcarriers. There exists a specific mathematical correlation between the frequencies of the multicarrier in the system hence the term "orthogonal" [1]. The basic concept of OFDM is to split a high-data-rate sequence into a number of low-rate sequences that are simultaneously transmitted by the transmitter over a number of subcarriers. The duration of the symbol is

increased for low rate parallel subcarriers hence the relative amount of dispersion caused by multipath delay spread is minimized. In order to evade Inter-symbol interference (ISI) arising from multipath effect, guard interval is added to isolate successive OFDM symbols. Multipath components interfering between successive symbols referred to as Inter-carrier interference (ICI), is avoided by cyclically extending guard interval and this is known as cyclic prefix. Thus, transformation of highly frequency selective channel into a large set of flat fading, narrowband channels, and non-frequency selective occurs. An implementation of Inverse Fast Fourier Transform (IFFT) or Discrete Fourier Transform (DFT) by an integrated circuit (IC) eliminates the requirement for the complete collection of separate transmitters and receivers. The Fast Fourier Transform (FFT) technique is used to remove collections of sinusoidal generators and coherent demodulation needed in parallel data systems and ensures cost effective implementation of the technology. Figure 1 is shows a block diagram of an OFDM system.



Fig. 1Block diagram of OFDM system

2.1 Advantages of OFDM

There are several advantages provided by OFDM that have made it a common scheme employed in a lot of high data rate wireless communication systems. Some of these are highlighted.

- Immune to selective to fading: A comparison of OFDM with single carrier systems shows that it is more unaffected to frequency selective fading because it splits the overall channel into multiple narrow-band channels.
- Exhibit interference resilient: In coming channel interference might be bandwidth limited and therefore has no effect on the entire the sub-channel. This alleviates the channel fluctuation.
- Efficient spectrum utilization: Employing overlapping orthogonal subcarriers that are closely-spaced makes possible the data transmission with low bandwidth channels and thus resulting in efficient use of frequency band (or spectrum) that is available.
- Inter-symbol interference (ISI) resilient: OFDM demonstrates high inter-symbol and inter-frame interference resilient. This is because of the fact that every sub-channel carries low data rate data stream.

- Narrow-band effects resilient: Using sufficient channel coding and interleaving facilitates recovery of symbols lost due to the frequency selectivity of the channel and narrow bad interference.
- Simpler channel equalization application: Generally, digital communication and spread spectrum communication channel equalization has to be employed across the whole bandwidth of the channel. As a result, channel equalization complexity increases. However, channel equalization in OFDM, requires only a single tap equalizer since multiple sub-channels are used in the scheme. Therefore, complexity of channel equalization is reduced in OFDM system.

2.2 Disadvantages of OFDM

Though OFDM has been largely used, certain disadvantages are still common with the scheme and required to be solved when considered for application.

- Carrier offset and drift sensitivity: Compared to single carrier system, OFDM exhibit more sensitivity to carrier frequency offset and drift.
- High peak to average power ratio (PAPR): Amplitude variation in time domain and relatively large range of dynamic characterise OFDM signals and this cause high PAPR. Since it is required that the RF amplifiers of transmitter be linear, the presence of high amplitude swings associated with OFDM signal results in nonlinear distortion and what these factors mean is that the amplifier cannot operate with a higher efficiency level.
- Complexity of receiver: OFDM receiver complexity increases with higher number of sub-channels.
- Computational complexity: Associated with OFDM system is computational complexity and this increases both at the transmitter and receiver with increasing subcarrier.

Having discussed OFDM, considering the advantages and disadvantages it associated with it, one obvious challenge is addressing the effect of PAPR. Several techniques have been proposed for PAPR reduction in OFDM signal. In some of the methods, the design has employed redundancy such as coding [4], [5], tone reservation [6], [7] or selective mapping (SLM) with explicit or implicit side information [8], [9]. However, an obvious effect of employing redundancy in minimizing PAPR is the reduction of transmission rate. The reduction of PAPR can also be achieved employing extended signal cancellation such as tone injection as in [6]. The resulting effect of this approach is increased power and computational complexity. Besides, knowing that conventional OFDM scheme itself suffers from complex computation, solving the problem of PAPR will require a technique that will not increased the already computational complexity related to it. Thus, in this paper a scheme to reduce PAPR is proposed with low computational difficulty.

III. SYSTEM DESIGN

In this section, the following subsections, mathematical definition of OFDM signal, mathematical definition of PAPR, Walsh-Hadamard Transform (WHT) scheme, log companding scheme, and proposed OFDM system are considered to arrive at goal of the study, which is to use Walsh-Hadamard Transform coded OFDM with Log compading (WHT-Log) algorithm to reduce the effect of PAPR of OFDM signal.

3.1 Mathematical Definition of OFDM Signal

Orthogonal Frequency Division Modulation (OFDM) consists of multiple carriers. Each carrier can be presented as a complex waveform given by:

$$\mathbf{s}_{c}(t) = \mathbf{A}_{c}(t)e^{j(\omega_{c}t + \phi_{c}t)} \tag{1}$$

where $A_c(t)$ is the amplitude of the signal $s_c(t)$, $\phi_c(t)$ is the phase of the signal $s_c(t)$.

The complex signal can be described by:

$$s_{s}(t) = \frac{1}{N} \sum_{n=0}^{N-1} A_{n}(t) e^{j(\omega_{n}t + \phi_{n}t)}$$
(2)

This is a continuous signal. Each component of the signal over one symbol period can take fixed values of the variables such that $\phi_n(t) \Rightarrow \phi_n, A_n(t) \Rightarrow A_n$. where n is the number of OFDM block.

At time interval T the signal is sampled by 1/T then (2) can be further expressed as:

$$S_{s}(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_{n}(t) e^{j[(\omega_{o}t + \omega \Delta n)kT + \phi_{n}]}$$
(3)

Assuming $\omega_0 = 0$, the signal becomes:

$$S_{s}(kT) = \frac{1}{N} \sum_{n=0}^{N-1} A_{n}(t) e^{j[(\omega \Delta n)kT + \phi_{n}]}$$

$$\tag{4}$$

The carrier signal is compared with standard Inverse Fourier Transform (IFT):

$$g(\mathbf{kT}) = \frac{1}{N} \sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right) e^{j[2\pi n k/N]}$$
(5)

Here, g(kT) is time frequency domain.

Both are equivalent if:

$$\Delta f = \frac{\Delta \omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau}$$
(6)

where τ is the symbol duration period

The OFDM signal can be defined by Fourier Transform. The Fast Fourier Transform (FFT) can be used to obtain frequency domain OFDM symbols and Inverse Fast Fourier Transform (IFFT) can be used to obtain time domain symbols. They can be written as:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j(2\pi/N)kn}$$
(7)

Inverse Fast Fourier Transform:

$$x(n) = \sum_{n=0}^{N-1} X(k) e^{j(2\pi/N)kn}$$
where, $0 \le n \le N - 1$.
(8)

3.2 Mathematical Definition of PAPR

Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio, or referred to

as PAPR, in some literatures, also written as PAR. It is usually defined as [10]:

$$PAPR = \frac{P_{peak-power}}{P_{avv-power}} = 10\log_{10} \frac{max[|\mathbf{x}_{n}|^{2}]}{E[|\mathbf{x}_{n}|^{2}]}$$
(9)

where P_{peak-power} represents peak output power,

 $P_{avv-power}$ means average output power. $E[\cdot]$ denotes the expected value, x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols X_k . Mathematically, x_n is expressed as in (8).

3.3 Walsh-Hadamard Transform Precoding

Precoding is a technique that involves multiplying the modulated data of each OFDM block in frequency domain by a precoding matrix **P** prior to IFFT process. The baseband modulated data stream is grouped into blocks of length $(N-N_p)$ symbols each [11]. An already established $N \times (N-N_p)$ precoding matrix **P** is used to multiply each block of symbols. The **P** matrix is given by [11]:

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,(N-N_p)} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,(N-N_p)} \\ \vdots & \vdots & \cdots & \vdots \\ P_{N,1} & P_{N,2} & \cdots & P_{N,(N-N_p)} \end{bmatrix}$$
(10)

where $P_{n,m}$ are the elements of the precoding matrix, N is the number of subcarriers, and $(N-N_p)$ denotes the data block length prior to precoding with $0 \le N_p < N$. The precoding matrix becomes $(N \times N)$ matrix when $N_p = 0$ and the rate loss reduces to zero [11]. The expression for the WHT precoding technique, which is a $(N \times N)$, is given by [11], [12]

$$WHT = H_1 = [1], H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix},$$

$$H_{2N} = \frac{1}{\sqrt{2N}} \begin{bmatrix} H_N & H_N\\ H_N & H_N^{-1} \end{bmatrix}$$
(11)

3.4 Proposed System

The proposed OFDM system combines WHT precoding technique with log companding in order to achieve reduced PAPR of OFDM signal. The block diagram of the system is shown in Fig. 2. It should be noted that only the transmit end is shown in the figure because the study is concerned with reducing PAPR which is a phenomenon that occurs at the transmitter.



For the purpose of simulation in MATLAB, the following parameters are utilized: modulation (QPSK), FFT size (varied: 16, 32, 64, 128, 256), spacing (15 kHz), bandwidth (1250 kHz), cyclic prefix (1/4 of FFT size), number of symbol (1000), sampling frequency (192 MHz), sampling period (192 μ s), maximum Doppler frequency shift (0.01 Hz).

IV. SIMULATION RESULT

The optimum parameter of k values ($k_1 = 1$, $k_2 = 1$) established in [2] for log companding were used in this paper. The number of subcarrier, N = 16, 32, 64, 128 and 256 were used to validate the effectiveness of the proposed scheme under varying N conditions. The simulation results are shown in Fig. 3 to 7 while the numerical performance evaluation is presented in Table 1. In order to determine the improvement achieved in percentage using the proposed scheme, (12) is used and it is given by:

$$\% PAPR_{improve} = \frac{PAPR_{original} - PAPR_{enhanced}}{PAPR_{original}} \times 100$$
(12)



Fig. 3 PAPR performance of OFDM signal (N = 16)



Fig. 4 PAPR performance of OFDM signal (N = 32)



Fig. 4 PAPR performance of OFDM signal (N = 64)



Fig. 5 PAPR performance of OFDM signal (N = 128)



Fig. 6 PAPR performance of OFDM signal (N = 256)

 Table 1 Performance analysis of WHT precoded

 OFDM system with log companding

	0		
	PAPR value (dB) at		
Number of	CCDF 10 ⁻³		improvement
subcarriers	Original	WHT – Log	
(N)	(dB)	(dB)	
16	7.964	4.775	40%
32	8.14	4.419	45.7%
64	8.885	4.285	51.8%
128	9.548	4.517	52.7%
256	9.961	4.602	57.8%

The simulations of peak to average power ratio (PAPR) performance of OFDM signal for different number of subcarriers (N) such that for N =16, 32, 64, 128, and 256, resulting PAPR of original OFDM signal was 7.964dB, 8.14 dB, 8.885 dB, 9.548 dB, and 9.961 dB respectively. With the introduction of WHT-log companding scheme, the PAPR of OFDM signal was significantly reduced to 4.775 dB, 4.419 dB, 4.285 dB, 4.517 dB, and 4.602 dB by a percentage improvement of 40%, 45.7%, 51.8%, 52.7%, and 57.8% for N = 16, 64, 128, and 256 respectively as shown in Table 1. The following observations were evident from the PAPR performance curves and the numerical analysis Tables 1: first, increase in the number of subcarriers can cause increase in PAPR value in OFDM system. Secondly, the use of log companding technique provides significant reduction of PAPR value of OFDM signal. Finally, by initially coding the OFDM signal with WHT precoding used in this paper and then introducing log companding can provide even more significant reduction of PAPR associated with OFDM signal. Generally, with respect to number of subcarrier, improvement in PAPR performance in OFDM system expressed in percentage increases with increase in number of subcarrier.

V. CONCLUSION

In this paper, the proposed WHT precoded OFDM system uses log companding technique to reduce the PAPR of OFDM signal. In this scheme, the data stream is fed into OFDM transmitter and it is initially precoded using WHT precoding matrix and then the log companding acts on it by compressing the coded OFDM signal and thereby eliminating the peaking effect of the signal that can push the transmitter amplifier into nonlinearity that can caused signal distortion. The simulation results revealed that for the different number of subcarriers considered, the proposed algorithm was able to offer significant reduction of PAPR in OFDM system. Generally, the WHT coded OFDM with log companding has shown improvement in the performance of PAPR.

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