State Variable Feedback Control of Data Centre Temperature

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I. INTRODUCTION

The number of users connected to the internet has largely increased in recent decades and this has led to huge data deposit in data centres of businesses and various public and private establishments. A data centre is a facility used for accommodating computer systems and other Information and Communication Technology (ICT) equipment. It has been referred in Gough et al. [1] as the information factory that shapes modern experience. In recent times, the amount of data required by companies to be processed is growing exponentially as a result of increasing demand for ICT support in every field of human endeavour. There are many data centre designs and infrastructure implementations such that each data centre reacts rather differently to variations in temperature [2]. Data centres are usually equipped with energy-efficient cooling system that serves as Computer Room Air Conditioner (CRAC). With increasing energy consumption and heat generation in data centres over the past few years, CRAC system improvement has been increasingly challenging [3]. A report cited by Hassan et al. [4] indicated that several data centres are still encountering challenges with high temperature zones generated within the data centres. Therefore, temperature control within data centre, and achieving maximum uptime and efficiency is necessary [5].

Several control techniques for temperature response performance improvement have been presented for data centres. Classical control systems involving the use of proportional integral and derivative (PID) controllers and feedback control schemes have be studied. Also, the use of intelligent algorithms such as fuzzy logic and neural network to optimize the parameters of PID controller has been proposed in [3],[6]. In order to improve the operational efficiency relative to existing PID controllers in a data-centre, Lazic et al. [7] used reinforcement learning (RL).

This paper aims to use an observer-based technique to control and estimate temperature in a data centre. Achieving this requires the use of mathematical expressions to model the dynamics of physical parameters in a data centre relating to temperature in the environment. That is, the thermal properties of data centre. Thus, the paper presents a state space model that is developed to study the performance characteristics of the internal or state variables of data centre temperature control system. Implementation is done in MATALB/Simulink environment. The ability to track a step input signal has been considered as the ability of the system to maintained desired computer room temperature and represents its efficiency. Also, the paper considers allowable temperature range of 18 to 27 °C specified by American Society of Heating, Refrigerating, and Air-Conditioning Engineers standard [8] for CRAC system of a data centre as a way of validating the effectiveness and ability of the proposed control system to achieved desired temperature.

II. STATE-SPACE THEORY AND OBSERVER MECHANISM

State variables can be used to represent the state of a plant or process as shown in Fig. 1. State variables are a set of variables whose knowledge and that of the input, together with mathematical expression (or dynamic equation) describing the system, determine the future state and output of the system.



Fig.1 Illustration of state variables

The response of a system in future is usually described by the system state variables provided the initial state, the inputs, and dynamic equations representing the system are known. Thus, the state space analysis of the input variables, output variables and state variables, which are the three variable of interest, can be used to formulate state space equation as shown in Fig. 1.

2.1 State-Space Equation

The mathematical expression relating the input variable(s), the state variables and the output variable(s) is known as state space equation. The development of state variable technique is made possible by the mathematical modeling of a system which, together with output, gives information on the state of the system variables at certain fixed points along the flow of signals. State variable design is directly a time domain scheme that provides a basis for modern control theory and system optimization [9]. Also, it serves as a very powerful technique for the analysis and design of linear, nonlinear, and time-invariant or time varying multiple input and multiple output (MIMO) system [9].The dynamic of linear system can be represented in state-space form given by:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where A, B, C, and D are the system matrix, the input (control) matrix, the output matrix and direct transition matrix.

2.2 Concept of Observer Mechanism

Observers are represented by mathematical or algorithmic equations with dynamic characteristics designed to incorporate the knowledge of system state and measurements for reconstruction of additional information that is related to assumed fundamental dynamic models. Observers represent an essential component part in the field of control and maintenance as a result of their ability to estimate non-measured states [10].

In feedback control system especially in temperature control system such as in refrigerators and air conditioners, sensors play a major role. However, using sensors to measure or detect internal non-measurable states of modelled system may not be cost effective or even unreliable; therefore the use of observers as alternative solutions is advantageous [10]. In addition to the estimation of system state, certain observers can estimate unknown inputs affecting the dynamics of the system such as disturbances.

Observers are largely employed in the estimation of system state x(t) as $\hat{x}(t)$. The widely used primary observers for estimating system states are Kalman filter and Luenberger observer in discrete and continuous time respectively [10]. Luenberger observer is widely used in the field of classical control due to its ability to estimate the states of system. It has a feedback loop to reconstruct system states in terms of measured outputs.

A full state-feedback control method with observer adopted in this study offers the following benefits over existing strategies:

- It provides the best performance compared to other controllers in terms of oscillation and settling time [11].
- It can address the problem of systems with timevarying state space representation [12], or systems with multiple operating conditions, as well as systems with multiple input and multiple output (MIMO) signals requirement [13].
- It also has flexibility properties of shaping the dynamics of the closed loop system to meet the desired specifications [14].
- When all the states of the plant cannot be measured (or can only be partially measured) or minimizing the number of measured states will be cost effective, designing an observer to estimate the unmeasured states of the plant becomes necessary.

III. SYSTEM MODELLING AND CONTROLLER DESIGN

This section presents the mathematical description of thermal process in data centre, both in transfer function and state-space representations as well as the MATLAB/Simulink model. It also covers the fundamental properties of system and design of a full state feedback controller with observer system.

3.1 State-Space Model Temperature in Data Centre

In the data-centre, the expected temperature is 20°C. The transfer function of open volume to temperature of a data centre can be represented as a system of second-order inertial and net delay process given by [3]:

$$G(s) = \frac{10}{(20s+1)(30s+1)}e^{-12s}$$
(3)

Solving (3) by defining the s-domain function as s = tf(s') in MATLAB, the transfer function is further expressed as given by :

$$G(s) = \frac{(10^{*}(\exp(-12s)))}{((20^{*}s+1)^{*}(30^{*}s+1))}$$
$$= \frac{10}{600 s^{2} + 50 s + 1} e^{-12s}$$
(4)

The detailed mathematical description and operation of the fan speed in Variable Air Volume (VAV) air condition system which was used in determining the air flux in the data-centre considered in this study can be found in Deng et al. [3]. Also note that 12 seconds delay is the same as 0.2 minute delay as used in the simulation analysis in this paper.

Equation (4) is transformed into state space-equation given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.0017 & -0.0833 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0167 \end{bmatrix} u(t)$$
(5)

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$
(6)
Equations (5) and (6) conform to the canonical form in (1)

Equations (5) and (6) conform to the canonical form in (1) and (2) and the Simulink model of the system is shown in Fig. 2. The system matrix, the control matrix and the output matrix are:



Fig. 2 System state-space model in Simulink

The MATLAB program for implementing the step response of the system without a controller is presented as follows:

It is important to consider certain basic properties that can impact on the control system such as system stability, controllability and observability. The stability of the openloop system is determined by computing the eigenvalues of system matrix A [15], which is equal to the poles of the transfer function. It was observed that all the poles are all negative. This means that the poles lie on the Left Hand plane (LHP) of the s-domain coordinates. Thus, the calculated poles are:

 $P_1 = -3.98 + 34.85i; P_2 = -3.98 - 34.85i.$ This means that the system is stable.

The controllability is applied to determine whether the system being studied is controllable. For linear time invariant (LTI) system, the system will be controllable if the controllability matrix C_r expressed by (7) is calculated such that the determinant is non-zero and of rank n (where n is the order of the system or number of state variables).

$$C_{r} = \begin{bmatrix} B & AB \end{bmatrix}$$
(7)
$$|C_{r}| = \begin{vmatrix} 0 & 0.0167 \\ 0.0167 & -0.014 \end{vmatrix} = -2.789 \times 10^{-4}, \text{ rank} = 2$$

Also, if the initial state $x(t_0)$, of a system can be determined based on what is known about the system input u(t) (simply represented as u) and the system output y(t), over some finite time $t_o < t < t_f$, the system is said to be observable. For LTI system, an observability matrix O_b , which must be of rank n with non-zero determinant is determined by:

$$\boldsymbol{O}_{\boldsymbol{b}} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C} \boldsymbol{A} \end{bmatrix}$$

$$|\boldsymbol{O}_{\boldsymbol{b}}| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, \text{ rank} = 2$$

$$(8)$$

3.2 Controller Design

This subsection is concerned with the study and application of pole placement technique. In pole placement technique, all reachable eigenvalues are assigned set of values inside stable region of s-plane. The poles of the closed loop system are expected to be moved by the pole placement technique in such a way that percentage overshoot of $\leq 5\%$ and rise time of ≤ 1 s is achieved. Full state feedback control and its implementation are carried out as follows.

3.2.1Design of Feedback Gain Matrix and Control Law For system represented in the canonical form as in (1) and (2), a state feedback control can be established (as shown in Fig. 3) based on a control action achieved by adding a feedback matrix \mathbf{K} to produce the control law u given by:



Fig. 3 Full state feedback control

Substituting the expression in (9) into (1), gives:

$$\dot{x} = Ax + Bu = Ax + B(-Kx + r)$$

$$= (A - B)x + Br$$
where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$.
(10)

The elements or gains of the state feedback matrix Knecessary to move the poles to the desired locations can be determined using direct method, which involves using the expression given by:

$$|\lambda \mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{K})| = \begin{bmatrix} \lambda & -1 \\ 0.017 + 0.0167 K_1 & \lambda + 0.0833 + 0.0167 K_2 \end{bmatrix}$$
(11)

Solving (11) gives the characteristics equation:

$$\lambda^{2} + (0.0833 + 0.0167 K_{2})\lambda + 0.0017 + 0.0167 K_{1} = 0 \quad (12)$$

Since the system is second order, it can be represented by the general characteristics equation of a second order system given by:

$$\lambda^2 + 2\varsigma \omega_n \lambda + \omega_n^2 = 0 \tag{13}$$

Choosing peak percentage value of $M_p = 5\%$ and substituting this value into the expression:

$$M_p = e^{-\pi\varsigma \left/ \sqrt{1-\varsigma^2} \right.} \tag{14}$$

gives the damping ratio ζ as 0.69. Assuming a settling time T_s of 1s, using the expression given by:

$$T_s = \frac{4}{\varsigma \omega_n} \tag{15}$$

The natural frequency response ω_n , was calculated to be 5.77 rads⁻¹. Substituting these values of the damping ratio and the natural frequency response into (13) gives:

$$\lambda^{2} + 7.963 \lambda + 33.293 = 0$$
(16)
Comparing equivalent values in Eq. (12) and (16) gives:

$$\boldsymbol{K} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 3329.13 & 787.97 \end{bmatrix}$$
(17)

Having determined the value of K the control law is given by u = -3329.13 787.97 x. The Simulink block diagram of the full state feedback control loop with the computed feedback gain matrix is shown in Fig. 4.



Fig. 4 Simulink model of system with feedback gain

The problem with using full state feedback only is that there is no guarantee of tracking desired setpoint value. Addressing this limitation, a gain mechanism is added in the forward path and is given as $K_f = 3329.13$, which is implemented in Simulink as shown in Fig. 5.



Fig. 5 Simulink model of full state feedback control with forward gain

3.2.2 Design of Observer System

It has been shown earlier that the plant under consideration is completely observable. Hence, Fig. 6 shows a full state feedback control system with observer mechanism. The observer states have "cap" above them, which signifies that states are estimated and not measured.

In full state feedback design, the choice of using an observer is optional though it is determined by inability to measure all the states. In any case, when all states are measurable, an observer may not be needed. However, performance cost may require the design of observer. The reason is that the number of sensors needed for measuring states required for full state feedback is reduced by observers [16].



Fig. 6 Full state feedback with observer

The mathematical theory of calculating and selecting observer gains is presented next. From Fig. 6, the observer state equation is given by:

$$\hat{\mathbf{x}} = A\hat{\mathbf{x}} + B\mathbf{u} + L(\mathbf{y} - \hat{\mathbf{y}}) |$$

$$\hat{\mathbf{y}} = C\hat{\mathbf{x}}$$
(18)

where $\mathbf{u} = \mathbf{K}_{f}\mathbf{r} - \mathbf{K}\hat{\mathbf{x}}$ and \mathbf{L} is the observer gain given by:

$$\boldsymbol{L} = \begin{bmatrix} L_1 & L_2 \end{bmatrix} \tag{19}$$

For an observer, the target is to reduce the error between the actual and the estimate states to zero.

$$error, e = x - \hat{x} \to 0 \tag{20}$$

$$\dot{e} = \dot{x} - \hat{x} = \mathbf{A}x + \mathbf{B}u - (\mathbf{A}\hat{x} + \mathbf{B}u + \mathbf{L}(y - \hat{y}))$$

= $(\mathbf{A} - \mathbf{L}\mathbf{C})(x - \hat{x})$
= $(\mathbf{A} - \mathbf{L}\mathbf{C})\mathbf{e}$ (21)

where y is the actual output temperature (T) and \hat{y} is estimated output temperature (\hat{T}) in degree Celsius.

Any time the eigenvalues (A - LC) are all negative, the error converges. In theory, the more negative the eigenvalues are, the faster the error reduces to zero. Also the states estimated by the observer converge to actual states more rapidly [16]. Since it is required of the dynamics of the observer to be much faster than the system itself, the poles should be placed at least five times farther to the left than the dominant poles of the system. A typical approach is to design the observer such that the eigenvalues are up to 5 to 10 times larger than desired closed loop poles. The Simulink block diagram of full state feedback with observer is shown in Fig. 7 and L is implemented using the MATLAB expression given by:

$$L = place(A', C', 10 * P)$$
Such that $L = \begin{bmatrix} 80\\ 123,030 \end{bmatrix}$. (22)



Fig. 7 Simulink model of implemented data centre temperature control system

IV. RESULTS AND DISCUSSION

The results obtained from the simulations conducted in MATLAB/Simulink environment for data centre temperature (DCT) control system are presented in this section.

4.1 Results

The step response (y0) to unit temperature input in degree Celsius of the data centre when controller has not been introduced is shown in Fig. 8. With the designed feedback gain added to the system to provide a full state feedback control mechanism, simulation was carried out and the response (y1) to unit input temperature is shown in Fig. 9. The response (y2) to unit input temperature with the introduction of forward path gain as part of the full state feedback (FSFB) controller is shown in Fig. 10. The step response plots of the controller (y2) and observer (y3) are shown in Fig. 11. The effectiveness of the estimation capacity of the observer is simulated in terms of the difference between the actual output temperature (controller response) and the estimated output temperature (observer response) that is $y - \hat{y}$ as shown in Fig.12. In order to validate effectiveness of the proposed controller, simulations were conducted considering various expected temperature in the data centre including 20°C and the step response performance plots are shown in Fig. 13. The summary of the numerical analysis of transient and steady state time domain performance characteristics of the various temperature responses is shown in Table 1.



Fig. 8 Temperature response (open loop)



Fig. 9 Temperature response (FSFBG only)



Fig. 10: Temperature response (FSFBG+FPG)



Fig. 11: Temperature response of controller/observer



Fig. 12 Estimated error output



Table 1 Step response performance analysis

Parameter	Responses			
	Uncompensated	FSFBG	*FSFB	FSFBO
	(y0)	(y1)	(y2)	(y3)
Rise time (minutes)	1.41	0.41	0.41	0.41
Overshoot	0	0.232	0.232	0.232
(%)				
Settling time	2.68	0.86	0.86	0.86
(1111111111)	10	0.0002	1	1
C)	10	0.0003	1	1
Steady state	-9	0.9997	0	0
CIIUI				

*Note: full state feedback gain (FSFBG) plus forward path gain (FPG) control system whose step response is represented as FSFBG + FPG in Figure 10 will simply be referred to as FSFB controller throughout the discussion.

Looking at Table 1, it can be seen that the rise time of the system was 1.41 minutes prior to the introduction of FSFB controller into the system but changed to 0.41 minutes when FSFB controller was introduced. This indicates 71.2% improvement on rise time of system due to the addition of FSFB controller into the system. The percentage overshoot was 0% when the system was without FSFB controller but changed to 0.232% due to the introduction of FSFB into the system. This shows that the FSFB compensated system performance has degraded compared to the uncompensated system in terms of

percentage overshoot. The settling time of the system was initially 2.68 minutes against 0.86 minutes for FSFB compensated system which indicated 68% improvement in the performance of system in terms of settling time. With respect to tracking of unit step temperature value, the uncompensated system yielded a final value of 10 °C (unable to track or achieve the desired unit step input temperature in the data centre) while the FSFB compensated system tracked the referenced unit step input temperature. The steady state error was -9 initially but changed to 0 when FSFB controller was added, which shows an improvement in steady state error performance.

Thus, it can be deduced from the results of the simulation analysis in Table 1 that the FSFB controller yielded improved transient and steady state responses because it was able to provide lower value of time domain parameters for system response to temperature in the data centre.

With the addition of observer to FSFB (FSFBO), the response (y3) performance was observed to be the same as that of FSFB as shown in Table 1. Thus the difference between the actual temperature and the estimated temperature was observed to be zero as shown in Fig. 12.

Furthermore, the effectiveness of the FSFB controller was validated by conducting simulation test considering various desired temperatures of data centre, whose values are in the range of 18 to 27 °C [3]. The simulation plots as shown in Fig. 13 indicated that the controller can provide desired temperature values in the data centre in accordance to establish standard in practice. The simulation results also revealed that the effect of delay was largely addressed by FSFB controller.

V. CONCLUSION

The paper has presented temperature control and estimation using full state feedback controller with observer mechanism in data-centre. The dynamic equation of open volume to temperature of a data-centre represented as a system of second-order inertial and net delay process was obtained. The transfer function model was transformed into equivalent state space model. The state space equation was represented by different features of Simulink used to model the system in MATLAB/Simulink environment. The performance of the system was studied in terms of stability, controllability, and observability. A full state feedback controller was designed and implemented in MATLAB/Simulink environment considering three steps namely, full feedback with no forward path gain, full state feedback with forward path gain, and full state feedback with an observer loop. Simulation results generally indicated that the proposed control system was capable of achieving perfect tracking and steady state error of zero.

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Biographies and Photographs



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