

# Algorithm for SNR Estimation In Rician Fading Channels

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## ABSTRACT

Orthogonal frequency division multiplexing provides strong robustness against inter carrier interference and also it provides good performance in frequency selective channels by adaptation techniques to estimate channel quality. An important parameter for calculating the channel performance is by signal to noise ratio (SNR). In this paper, we propose a novel for SNR estimation using synchronization pREAMbles for rician fading channels. The pREAMbles of periodic structure are used for efficient calculation for SNR estimation. The proposed estimator performance is compared with previous estimators and the proposed estimator is robust against frequency selectivity in the channel.

**Keywords** – Inter Symbol Interference (ISI), Channel State Information (CSI), Cramer-Rao Bound (CRB), Fast Fourier Transform (FFT), PS Estimator.

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## I. INTRODUCTION

OFDM divides the broadband channel into many narrowband sub channels in order to provide less attenuation across each sub channel. So, OFDM is a multi-carrier modulation technique. OFDM operates on the orthogonalization of the sub channels. Inorder to produce low complexity in performing orthogonalization of sub channels we prefer Fast Fourier Transform (FFT). As the high data rates are transmitted in serial form those are converted into many parallel low rates and uses different modulation techniques for each sub channel.

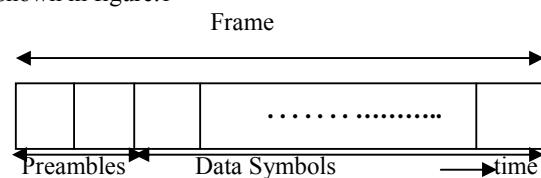
The main aim of designing of OFDM system is to gain an advantage in frequency selective channels by variable parameters in transmission like bandwidth, coding/data rate, power to protect and to provide improvement in efficient utilization of power and bandwidth according to the behavior of the channel at the receiver. In order to have improvement at the receiver, an exact and efficient signal-to noise-ratio (SNR) estimation algorithm is needed.

SNR estimators are used to find the performance of different systems. It also used to improve the performance of the channel. The estimator used to derive SNR from the modulating, sampled, data-bearing received signal. The data used may be known or unknown. If the known data is

considered from the training sequences provided by the synchronization and equalization bits of the data.

## II. SYSTEM DESCRIPTION

The system aim is to find the best estimator for SNR in a digital receiver in an OFDM system. Normally the estimation of the SNR is done by averaging the received signal properties. In wireless OFDM systems, data is transmitted in the form of frames. And the frame structure is as shown in figure.1



**Fig.1: Frame Structure.**

In the above figure, the data symbols in frame are added with several training symbols of known data used for the synchronization and equalization purposes. And these pREAMbles are also used to find the channel estimation. Let us consider the frame structure which consists of pREAMbles where each preamble composed of Q modulated subcarriers. Here  $x(p,q)$  denotes the data symbol which is complex with qth subcarrier in  $p^{th}$  preamble, where  $p=0,1,\dots,P-1$  and  $q=0,1,\dots,Q-1$ . The

magnitude of the modulated subcarrier is assumed to be unity, i.e., magnitude of  $|X(p,q)|^2=1$ , and this is the regular assumption for the OFDM systems which utilizes the preambles composed of BPSK and/or QPSK modulated subcarriers.

As we consider, the performance of the estimation only in frequency domain, so the received signal contains only the characterization in frequency domain of frequency selective channels. And we also assume that the data received at the receiver is perfectly synchronized with the transmitted signal as we are using Fast Fourier Transform (FFT) for the transmission. The received signal on the  $q^{th}$  subcarrier in  $p^{th}$  preamble can be given by,

$$Z(p, q) = \sqrt{P_s} X(p, q) Y(p, q) + \sqrt{P_n} N(p, q) \quad (1)$$

Where  $N(p,q)$  is the zero -mean sampled complex noise with unit variance ,  $P_s$  is the signal power and  $P_n$  is the noise power added in the channel on individual subcarriers, and  $Y(p,q)$  is the frequency response of the channel and is given by,

$$Y(q) = \frac{2(K+1)q}{\Omega} \exp\left(-K - \frac{(K+1)q^2}{\Omega}\right) I_0\left(2\sqrt{\frac{K(K+1)}{\Omega}}q\right) \quad (2)$$

where ,  $q \geq 0$

For Rician fading, $I_o$  denotes the  $0^{th}$  order modified Bessel function of the first kind. And, the total power  $\Omega=v^2 + 2\sigma^2$  which acts as a scaling factor to the distribution. And,

$$v^2 = \frac{K}{1+K} \Omega \quad \& \quad \sigma^2 = \frac{\Omega}{2(1+K)}$$

Where, K is the rician factor which is constant always and K always greater than or equal to 0 i.e., ( $K \geq 0$ ).

Our initial assumption is that frame in the channel is constant, since the considered estimator for the SNR estimation uses the adaptive transmission. We also assume that the average SNR and the subcarrier SNR estimation are valid for all data-bearing information of the OFDM symbols in the frame. As [5] shows, the average SNR for  $p^{th}$  preamble received in OFDM system and is expressed as,

$$P_{ASNR}(p) = \frac{E\left\{\frac{1}{Q} \sum_{q=0}^{Q-1} |\sqrt{P_s} X(p,q) Y(q)|^2\right\}}{E\left\{\frac{1}{Q} \sum_{q=0}^{Q-1} |\sqrt{P_n} N(p,q)|^2\right\}} \quad (3)$$

$$= \frac{P_s}{P_n}$$

Due to adaptive transmission, time factor p is omitted during the estimation process, i.e.,  $Y(p,q)$  is represented as  $Y(q)$ . And,  $\sum_{q=0}^{Q-1} |Y(q)|^2 = Q$  is satisfied, when the SNR of the qth subcarrier is given by,

$$P_{sc}(p, q) = \frac{E\left\{|\sqrt{P_s} X(p,q) Y(q)|^2\right\}}{E\left\{|\sqrt{P_n} N(p,q)|^2\right\}} \quad (4)$$

$$= \frac{P_s |Y(q)|^2}{P_n} = P_{ASNR} |Y(q)|^2$$

### III DIFFERENT ESTIMATORS FOR SNR

#### 1. MMSE Estimator:

MMSE Estimator algorithm in OFDM systems is based on the orthogonality between the error estimation and the channel response estimation and is expressed as,

$$(Z(q) - \bar{Y}(q)X(q))(\bar{Y}(q)X(q)) * = 0, q=1,2,...,Q.$$

Where,  $\bar{Y}(q)$ represents the  $Y(q)$  estimation and  $*$  represents the conjugate operation. The average MMSE SNR estimation is expressed by [2],

$$\bar{P}_{ASNR,M} = \frac{\bar{P}_{SM}}{\bar{P}_{NM}}, \quad (5)$$

Where, transmitted signal power in MMSE is given by,

$$\bar{P}_{SM} = \left| \frac{1}{Q} \sum_{q=0}^{Q-1} Z(q) X(q)^* \right|^2$$

And, noise power in MMSE is given by,

$$\bar{P}_{NM} = \frac{1}{Q} \sum_{q=0}^{Q-1} |Z(q)|^2 - \bar{P}_{SM}$$

#### 2.Boumard's Estimator:

In [6] , the second-order moment based SNR estimator was proposed by Boumard. In slow varying channel in the OFDM systems, this estimator is used. This estimator can be used for time as well as frequency domain. This estimator gives the results when the inputs and outputs are in the form of  $2\times 2$  matrix. This also called as MIMO estimator.

In [5], Ren estimator used to derive the corresponding SISO version of Boumard's estimator by assuming that the channel used is time-invariant and also uses two preambles which are identical for estimating SNR, i.e.,  $p=0,1$  only and  $X(p,q)=X(1,q)=X(q)$ , for  $q=1,2,\dots,Q$ . Average SNR estimation can be expressed by,

$$\bar{P}_{ASNR} = \frac{\bar{P}_{SB}}{\bar{P}_{NB}} \quad (6)$$

Where, transmitted signal power in Boumard's is given by,

$$\bar{P}_{SB} = \frac{1}{Q} \sum_{q=0}^{Q-1} |\bar{Y}(q)|^2$$

And, noise power in Boumard's is given by,

$$\bar{P}_{NB} = \frac{1}{4Q} \sum_{q=0}^{Q-1} |X(q-1)(Z(0,q) + Z(1,q)) - X(q)(Z(0,q-1) + Z(1,q-1))|^2$$

The channel frequency response is given by,

$$\bar{Y}(q) = \frac{X(q)^*}{2} (Z(0,q) + Z(1,q)) \quad (7)$$

Where,  $\bar{Y}(q)$  is the LS(least Square)estimate of  $Y(q)$  mean over two preambles. By using  $\bar{Y}(q)$ , SNR estimation of  $q$ th subcarrier is given by,

$$\bar{P}_{sc}(q) = \frac{|\bar{Y}(q)|^2}{\bar{P}_{NR}} \quad (8)$$

### 3.Ren's Estimator:

The Ren's estimator overcome the drawback of previous estimator i.e., high sensitivity to frequency selectivity. In [5], this estimator provides more accurate estimation for second-order SNR estimation and is robust to the frequency selectivity and we consider the arrangement of the preamble as in Boumard's estimator. The average SNR can be expressed as,

$$\bar{P}_{ASNR} = \frac{\bar{P}_{SR}}{\bar{P}_{NR}} \quad (9)$$

Where, noise power for Ren's estimator is given by,

$$\bar{P}_{NR} = \frac{4}{Q} \sum_{q=0}^{Q-1} \left\{ \text{Im} \left[ \frac{z(0, q) X^*(0, q) \bar{Y}^*(q)}{|\bar{Y}^*(q)|} \right] \right\}^2$$

And, signal power for the Ren's estimator is given by,

$$\bar{P}_{SR} = \frac{1}{Q} \sum_{q=0}^{Q-1} |Z(0, q)|^2 - \bar{P}_{NR}$$

Here  $\bar{Y}(q)$  is the channel frequency response and is expressed as shown in equation (7). In this estimator, the performance of the estimator is independent of the frequency response of the channel but to find the average SNR estimation using estimated channel state is used. And, SNR per  $q$ th subcarrier is given by equation (8).

### IV PROPOSED ESTIMATOR

The proposed estimator uses the periodically the subcarriers for calculation of average SNR per subcarrier. The estimator is named as periodic sequence estimator(PS estimator). The main idea to design this estimator is to provide synchronization for both time domain and in frequency domain using time based periodic preamble structure as explained in [4]. To cover a wider range of frequency, a preamble structure is proposed with  $I$  identical parts and each parts contains  $Q/I$  samples from [3] as shown in fig:2(i). And fig:2(ii) shows the respective frequency domain representation. As we assumed that  $I$  dividing  $Q$ , so that it is represented as

$$Q_k = Q/I ; \text{ Which is an integer}$$

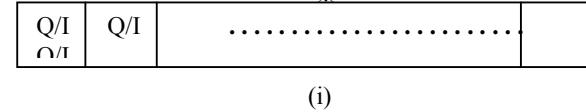
Here from starting subcarrier onwards each subcarrier is modulated with a QPSK signal  $X_k(m), 0, 1, \dots, Q_k - 1$  with  $|X_k(n)| = 1$ . The subcarriers  $Q_z = Q - Q_k = \frac{(I-1)}{I} Q$  are going to be nulled. To have a total power for all symbols within the preamble, then the power is scaled by a factor  $I$  giving a total transmitted power of  $P_s I$  in the loaded subcarriers. Represent  $q=mI+i, m=0, 1 \dots I-1$

The transmitted signal on the  $q^{th}$  subcarrier is given by,

$$X(q) = X(mI + i) = \begin{cases} X_k(m), & i = 0 \\ 0, & i = 1, 2, \dots, I-1 \end{cases} \quad (10)$$

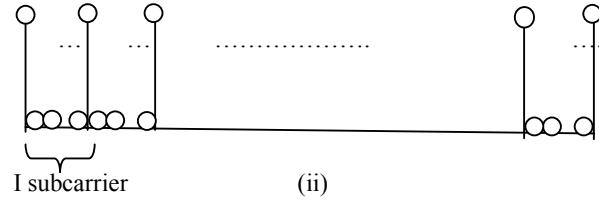
Then the received signal is given by,

$$Z(q) = Z(mI + i) = \begin{cases} Z_k(m), & i = 0 \\ Z_z(mI + i), & 1, 2, \dots, I-1 \end{cases}$$



(i)

$$\sqrt{P_s I} \quad \sqrt{P_s I} \quad \sqrt{P_s I} \quad \dots \quad \sqrt{P_s I} \quad \sqrt{P_s I}$$



(ii)

**Fig: 2. Structure of preamble in (i) Time domain and (ii) Frequency domain.**

Where

$$Z_k(m) = \sqrt{P_s I} X_k(m) Y(m) + \sqrt{P_N} N(m) \quad (11)$$

Which denotes the received signal on signal carrying subcarriers, and

$$Z_z(mI + i) = \sqrt{P_N} N(mI + i) \quad (12)$$

This denotes the received signal on zero signal carrying subcarriers which contains only noise signal. The empirical second order moment for signal carrying subcarriers is given by,

$$\bar{R}_{2,k} = \frac{1}{Q_k} \sum_{m=0}^{Q_k-1} |Z_k(m)|^2 \quad (13)$$

And the expected value is given by,

$$E[\bar{R}_{2,k}] = \frac{1}{Q} \sum_{k=0}^{Q-1} E[|Z_k(m)|^2]$$

$$\begin{aligned} &= \frac{P_s I}{Q_k} \sum_{m=0}^{Q_k-1} E[|Y(m)|^2] + \frac{P_N}{Q_k} \sum_{m=0}^{Q_k-1} E[|N(m)|^2] \\ &= P_s I + P_N \end{aligned}$$

Similarly, the empirical second moment of the received signal for zero signal carrying subcarriers,

$$\bar{R}_{2,z} = \frac{1}{Q_k(I-1)} \sum_{m=0}^{Q_k-1} \sum_{i=0}^{I-1} |Z_z(mI+i)|^2 \quad (14)$$

And the expected value is given by,

$$\begin{aligned} E[\bar{R}_{2,z}] &= \frac{1}{Q_k(I-1)} \sum_{m=0}^{Q_k-1} \sum_{i=0}^{I-1} E[|Z_z(mI+i)|^2] \\ &= \frac{P_N}{Q_k(I-1)} \sum_{m=0}^{Q_k-1} \sum_{i=0}^{I-1} E[|N(mI+i)|^2] \\ &= P_N \end{aligned}$$

Hence, the average SNR  $P_{ASNR}$  can be estimated by forming,

$$\begin{aligned} \bar{P}_{ASNR} &= \frac{1}{I} \frac{\bar{R}_{2,k} - \bar{R}_{2,z}}{\bar{R}_{2,z}} \\ &= \frac{1}{I} \left( (I-1) \frac{\sum_{m=0}^{Q_k-1} |Z_k(m)|^2}{\sum_{m=0}^{Q_k-1} \sum_{i=0}^{I-1} |Z_z(mI+i)|^2} \right) \quad (15) \end{aligned}$$

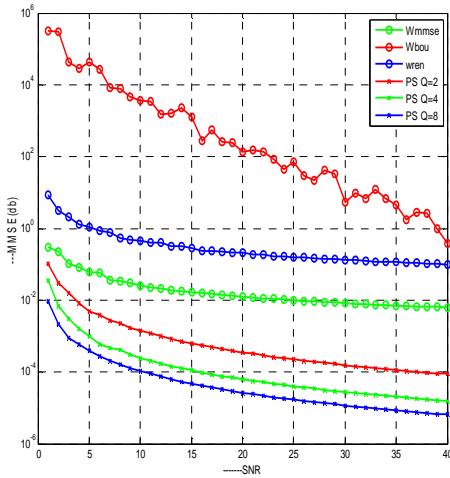


Fig: 3. NMSE of the Average SNR in AWGN channel.

For the large numbers of  $\bar{R}_{2,k}$  and  $\bar{R}_{2,z}$  are unbiased estimators of  $P_s I + P_N$  and mean noise power  $P_N$ , respectively.

As compared to the previously described estimators, the PS estimator does not require any knowledge of the symbols transmitted on signal carrying subcarriers. But the arrangement of signal carrying and zero signal carrying subcarriers at the receiver is needed to estimate the SNR. The estimation of the channel  $\bar{Y}(m), m = 0, 1, \dots, Q_k - 1$ , are presented for the signal carrying subcarriers.

When we use this kind of representation to transmit power on each signal carrying subcarriers which ultimately increases the accuracy due to increase of the factor I. By using Linear or DFT based interpolation as in [7]. We can estimate channel for zero signal carrying subcarriers.

The equation (8) is used to estimate the SNR on the  $q^{th}$  subcarrier and the noise power is estimated from the equation (14). Hence, if we increase the number of parts that structure divided,  $Q_k$  improves the accuracy of the noise power and the sensitivity of SNR estimation per subcarriers also increases to selectivity of the frequency because the performance increase by using interpolation on zero signal carrying subcarriers while channel estimation.

From the designing part of view and coming to implementation also, the proposed PS estimator has less complexity than Boumard's and Ren's estimators. To estimate average SNR, Boumard's estimator needs 5Q multiplications and 2Q additions per estimate, respectively. To estimate average SNR by Ren's estimator, it needs 4Q multiplications and 3Q additions per estimate. But, for the PS estimator we required only Q multiplications and Q additions per estimation. This PS estimator is having high bandwidth efficiency and it is using preambles rather than of Boumard's and Ren's estimator.

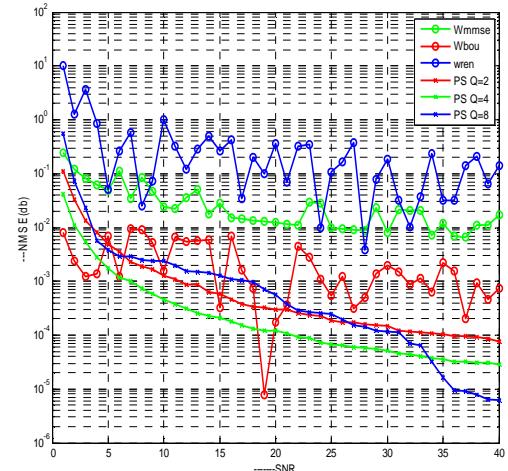


Fig: 4. NMSE of the average SNR for channel (b) and (c)

## V SIMULATION RESULTS

The PS estimator is compared with the previous estimator like MMSE, Ren's, and Boumard's estimators based on their performances using Montecarlo simulation. For the OFDM systems, the parameter specifications are taken from the WIMAX. Where Q is taken as 256 subcarriers and cyclic prefix length of 32 samples [8].

Performance is observed from three different channels (a) AWGN channel (b) a tie-invariant fading channel with delay spread  $t = 2$  samples (c) a time-invariant fading channel with delay spread  $t = 10$  samples. The considered channels parameters are taken from [6]. The number of trials performed independently is taken as  $Q_t = 100000$  to improve the results of the estimation. The performances is

considered in the form of Normalized Mean Square Error (NMSE) for the estimated average SNR values by,

$$NMSE_{ASNR} = \frac{1}{Q_t} \sum_{j=1}^{Q_t} \left( \frac{\bar{P}_{ASNR,j} - P_{ASNR}}{P_{ASNR}} \right)^2 \quad (16)$$

Where,  $\bar{P}_{ASNR,j}$  denotes the estimation of average SNR in the  $j^{th}$  trial, and  $P_{ASNR}$  denotes actual value. If we performance the NMSE to estimate SNR per subcarrier by using,

$$NMSE_{SC} = \frac{1}{Q_t} \sum_{j=1}^{Q_t} \sum_{q=0}^Q \left( \frac{\bar{P}(q)_j - P(q)}{P(q)} \right)^2 \quad (17)$$

Where,  $\bar{P}(q)_j$  gives the estimation of  $P(q)$  in the  $j^{th}$  trial. As the MMSE and proposed PS estimator algorithm uses only one preamble for the estimation process, while a Boumard's and Ren's estimator uses two preambles for the estimation. We are going to perform the evaluation for three different cases of preamble's repeated parts i.e.,  $L=2, 4$  and  $8$ .

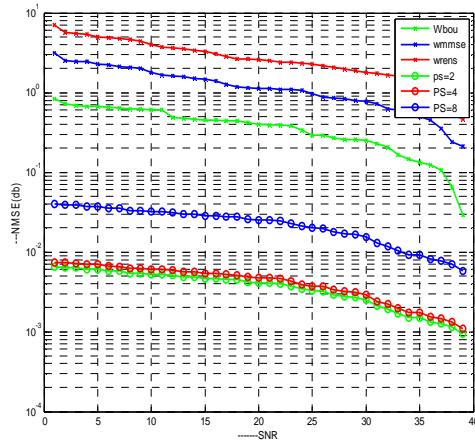


Fig.5: NMSE of the average SNR per subcarrier in channel (b)

#### AWGN Channel:

Fig. 3 shows the AWGN channel for  $NMSE_{ASNR}$  estimation. For the absolute performances of the estimators, they are compared with the Cramer-Rao Bound (CRB) and it is a lower bound unbiased estimator, see [9]. The Normalized CRB ( $N_{CRB}$ ) for OFDM signal with Q QPSK modulated subcarriers in AWGN channel. And is expressed as,

$$N_{CRB} = \frac{1}{Q} \left( \frac{2}{P_{ASNR}} + 1 \right) \quad (18)$$

The best performance of the  $NMSE_{ASNR}$  curve is undistinguished from the Normalized Cramer-Rao Bound ( $N_{CRB}$ ) for the NMSE estimator from equation (18).

Compared to Ren's and PS estimator, for 10db smaller value of average SNR the performance is decreased for Boumard's estimator. And if the 20db greater value is taken than the performance increased for Boumard's estimator compared to Ren's and Ps estimator for  $L=2$ . If we going on increasing the number of repeated parts to 4 and 8, the

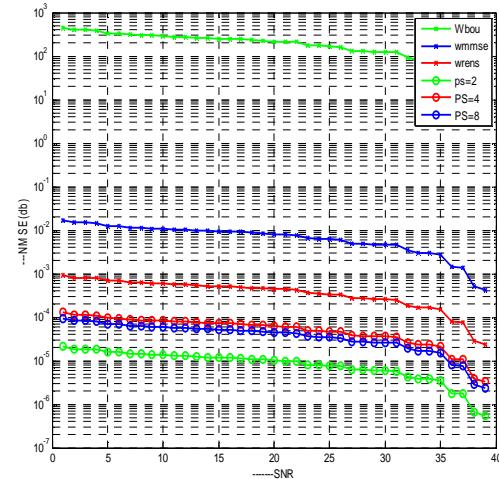


Fig.6: NMSE of the average SNR per subcarrier in channel (c)

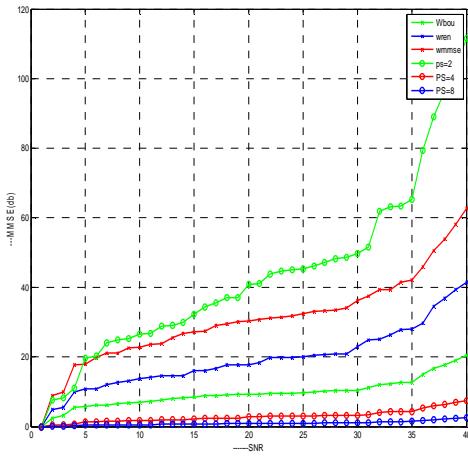


Fig.7: Mean of the estimated average SNR in channel (b)

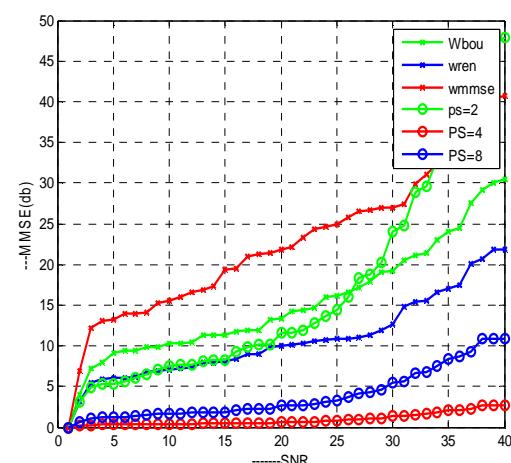


Fig.8: Mean of the estimated average SNR in channel (c)

performance will be close to the  $N_{CRB}$ . We can explain it by considering the noise power estimation (14), if more subcarriers used to estimate average SNR then the transmitted signal power increased by the scaling factor L on signal carrying subcarriers, and gives more accurate estimation in (13).

### B. Time – Invariant Frequency Selective Channel.

Fig.4 compares the performance of the  $NMSE_{ASNR}$  of considered estimators in time invariant frequency selective channels (b) and (c).

As the Ren's estimator performance doesn't depends on the frequency selectivity, while Boumard's estimator is highly sensitive to channel selectivity, so the performance varies. PS estimator responds to the channel (b) moderately but the performance is same as that of AWGN channel for all values of I, outperforming Ren's and Boumard's estimators. PS estimator does not respond accurately when the value of I increases for channel (c) as it is strongly selective to the biased in the channel with strong selectivity and the bias value normally is 1.82dB over the entire SNR estimation range. The normalized MSEsc performance is shown in Fig.5&Fig.6. And Fig.5 represents the estimated SNR per subcarrier in the channel (b). Fig.6 represents the estimated SNR per subcarrier in the channel(c). If we considered all estimators when the performance is bad then the SNR estimated gives low values. If we pilot subcarriers in the data symbols then the performance will be improved further. If the value of SNR is high, then the channel estimates stop changing with inputs and NMSEsc behaves and equal to  $NMSE_{ASNR}$ . The PS estimator performance is approximately improved from the Ren's estimator but expect in the case of I=8 because its performance depends on channel selectivity. And by using interpolation we can find or expect the behavior of the channel. Fig.7& Fig.8 shows the mean of the average estimated SNR in both channel (b) &(c).

### VI CONCLUSION

In this paper, we are proposing a algorithm for SNR estimation in Rician channel in wireless OFDM systems. Here in this novel algorithm we will reuse the synchronization Training symbols for the estimation of the SNR. Here all thisestimated values of SNR for proposed estimator. Compared to k=1,3,5,7&10, for k=5,the proposed estimator gives the best results.

The considered preamble structure uses time domain for reduction of additional overhead on the transmitted OFDM frame. The performance of the proposed estimator is improved by increasing the number of repeated parts by nulling the subcarriers on unwanted portion. And this also increases the estimator sensitivity to frequency selectivity. The proposed estimator favors the OFDM systems in complexity, bandwidth efficiency compared to the previous SNR estimator using preambles in given literature.

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